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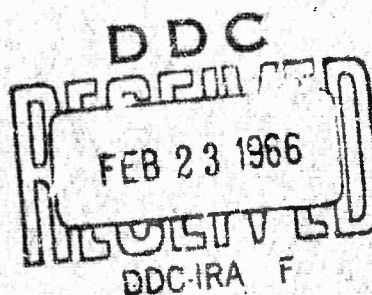
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GEOMETRY AND PATTERNS
OF LARGE APERTURE SEISMIC ARRAYS

R. T. LACOSS

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ABSTRACT

A study of possible configurations for a Large Aperture Seismic Array has been completed. An array having 21 identical subarrays located on three concentric circles has been found to yield the most satisfactory pattern in wave number space of all the configurations tested. Patterns for some alternative placements of subarrays, including that of the experimental LASA in Montana, have been included in this report.

This study of patterns in wave number space has yielded the suggestion that a LASA having a diameter of 200 km should be composed of subarrays from 10 to 15 km in diameter. Such an increase of subarray size above the 7 km diameter subarrays of the experimental LASA would require the use of less regular subarray geometries than those which have been used in Montana.

A sensitivity function for patterns has been developed. This function can be used to predict the change in patterns which might result from changes in seismometer or subarray positions. Since the sensitivity function predicts possible changes in patterns, it can be used to set bounds upon changes in an array which can be made without severely changing the pattern. The tight bounds imposed by the sensitivity function can be relaxed if the pattern resulting from any anticipated change in position is actually computed. Since very little computer time is required to compute a pattern, this procedure is highly recommended.

"We will have rings and things and fine array."
Shakespeare, Taming of the Shrew, Act II,
Scene 1.

I. INTRODUCTION

An experimental Large Aperture Seismic Array (LASA) has been deployed over a large .tion of southeast Montana. The extent of this array is about an order of magnitude greater than that of any previous instrument used in the study of seismic signals. The vertical seismometers of the experimental LASA are spread over a region which can be contained in a circle having a radius of approximately 100 km. The array consists of 21 subarrays each containing 25 seismometers spread over a circular region with a radius of 3.5 km.

Figure 1 shows the placement of the 21 subarrays of the experimental LASA. The basic configuration of seismometers in each subarray is shown in Fig. 15B. In many subarrays of the experimental LASA the basic subarray geometry was rotated up to several degrees. Thus subarray configurations are essentially identical to within rotations. References 6 and 9 present the various reasons which led to the installation of an array of subarrays and to the particular configuration which has been installed.

Studies are currently in progress, primarily at M. I.T., Lincoln Laboratory, to determine how best to utilize the voluminous data from the LASA for the detection of teleseismic events and the discrimination between natural and man-induced tremors. Commensurate with these goa.s the array is also being used to study aspects of earth structure, propagation phenomena, and seismic noise properties. All of this research

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deals with the use of data obtained from an already existing array. The prime goal is to determine the ability of a LASA to monitor underground nuclear tests.

If it is determined that a LASA is sufficiently effective for seismic detection and discrimination to warrant installation of several more such arrays, it may prove desirable for various reasons to consider geometries different from that of the experimental LASA. It is with this contingency in mind that this report has been prepared. Several families of possible array geometries have been examined to find a few of the most promising ones. Some measures of the sensitivity of array properties with seismometer positions have also been obtained.

The same computer programs used to study and evaluate array geometries for this report could also be used to evaluate alternative configurations which might be suggested. Also, these programs might well be useful before installation to study possible detrimental or beneficial effects resulting from perturbations of an intended array geometry. Such perturbations could result from constraints imposed by local topological conditions, land availability, or other factors.

II. REVIEW OF BASIC FACTS ABOUT ARRAY PATTERNS

The effectiveness of an array and associated data processing scheme for the reduction of seismic data can be measured by the response, in the three-dimensional frequency wave number space, of the array and associated data reduction scheme. References 3, 4, 5, 6, 8, and 10 are six of many in which the representation of seismic signals in frequency wave number space is discussed and the interpretation of data processing methods as filters in that space is explained.

The study of array patterns is the study of filter characteristics in frequency wave number space for a most simple kind of data processing. Specifically, if the array contains N seismometers, then a composite trace can be formed simply by adding the N traces and dividing by N. The pattern of the array from which the N traces are obtained is the magnitude of the gain of the array in frequency wave number space when such a straight sum processing technique is used.

If seismometer responses are frequency independent and no frequency dependent gains are inserted before summing, then the gain of the processing scheme is independent of frequency. Under these conditions only the response in two-dimensional wave number space need be considered. If a single frequency filter operates upon the sum waveform or identical filters operate upon every one of the individual traces, then it is still valuable to consider only response in wave number space assuming no filtering before summing. The gain at any point in frequency wave number space is the gain in the plane where frequency is zero, multiplied by a function of frequency only. The response in the plane where frequency is zero is the response in two-dimensional wave number space.

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This report deals only with the gain in wave number space obtained when straight sum processing is employed. Complex processing schemes which require explicit consideration of frequency as well as wave number in order to be characterized are beyond the scope of this paper.

Different patterns can be obtained as a function of array geometry without changing the data processing method. A comparison of the array geometries can then be made on the basis of the patterns which represent the signal processing ability of the array. Very roughly, one wishes to synthesize patterns which have a maximum gain at the origin in wave number space and which are uniformly low in some region around the origin of wave number space. The extent of the region over which the array must have low gain depends upon the characteristics of seismic noise.

The pattern as described above is in fact the magnitude of a two-dimensional Fourier transform of the array geometry. Thus the pattern of an array with vertical seismometers at positions $\{x_j, y_j : j = 1, \ldots, N\}$ is

$$|H(k_x, k_y)| = \frac{1}{N} \left| \sum_{j=1}^{N} \exp\left[-i\,2\pi\,(k_x x_j + k_y y_j)\right] \right| \tag{1}$$

where k_x and k_y are respectively east and north components of horizontal wave number. $H(k_x, k_y)$ can be interpreted as the two-dimensional transform of an impulsive function with unity weights on each impulse. The sifting property of the impulse function changes the integrals expressing the transform into the above sum over seismometer positions.

As mentioned in the introduction, the experimental LASA in Montana is an array of roughly identical subarrays. The pattern of such an array can be approximated by

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the pattern of a compound array. A compound array is one which is an array of identical subarrays. A compound array has the property that the pattern can be expressed as the product of the pattern of a subarray and the pattern of a configuration having a single seismometer located in the place of the subarrays. To see this, let x_{pq} and y_{pq} be the x and y coordinates of the pth seismometer in the qth subarray. Let

$$x_{pq} = x_p^o + x_q$$

and

$$y_{pq} = y_p^o + y_q$$

where x_p^o, y_p^o locate the center of the pth subarray and x_q, y_q locate the seismometers of the subarrays with respect to their centers. Let P be the number of subarrays and Q be the number of elements in the subarray. Then

$$H(k_x, k_y) = \frac{1}{PQ} \sum_{p=1}^{P} \sum_{q=1}^{Q} \exp\{-i2\pi[k_x(x_p^o + x_q) + k_y(y_p^o + y_q)]\}$$

$$= \left[\frac{1}{P} \sum_{p=1}^{P} \exp[-i2\pi(k_x x_p^o + k_y y_p^o)]\right] \left[\frac{1}{Q} \sum_{q=1}^{Q} \exp[-i2\pi(k_x x_q + k_y y_q)]\right].$$

Clearly the magnitude of the product of the right is the product of the magnitudes and the two terms of the product are the patterns of the array centers and the subarray respectively. Henceforth we shall use the terms full array pattern, array pattern, and subarray pattern. Full array pattern and array pattern are used to distinguish between

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the patte.. generated using all seismometers in an array and the pattern generated by the centers of the subarrays in a compound array, respectively. Subarray pattern of course refers to the pattern of a subarray in a compound array.

In this report a direct confrontation of the full array problem has been avoided. Only idealized compound arrays having the factorization property just described have been considered. Subject to any minor positioning errors which occur during installation and even to subarray rotations such as those of the experimental LASA this separate consideration of array and subarray patterns rather than full array patterns has been found to be a valid procedure, as will now be demonstrated.

Figure 2 shows the array pattern, evaluated along a line in wave number space, of the actual subarray positions of the experimental LASA. Figure 3 shows a similar plot of the full array pattern which has been obtained using all 525 seismometers. Note that the patterns are very similar for wave numbers less than .05 cycles/km and that for larger wave numbers the full array pattern has smaller sidelobes than the array pattern. The location of subarrays in the experimental LASA was fixed assuming that the subarray pattern would multiply the array pattern to yield the full array pattern. The idea was to pick a subarray and array so that this product would yield a full array pattern having all but the nearest lobes of the array pattern significantly reduced. Figures 2 and 3 clearly show that such a philosophy can yield a satisfactory full array pattern even when individual subarrays differ from each other by being rotated several degrees.

The experimental LASA cannot have a pattern which is exactly the product of the array and subarray patterns since the different subarrays have patterns differing by rotations. However, such an approximation, using the pattern of one subarray, is quite good for wave numbers in the main lobe of the subarray pattern since the main lobe changes very little under rotation. For larger wave numbers the approximation breaks down but the full array pattern still has the required low sidelobes. Figure 17B, if the wave number scale is multiplied by 200/7, is a cross section of one of the sub-array patterns for the experimental LASA. The plots in Figures 2 and 3 extend only about as far in wave number as the main beam of a subarray.

Straight sum patterns described above will have a main lobe located at the origin in wave number space. Thus the filter operation will pass signals which have vertical incidence upon the array and thus have infinite horizontal phase velocity, or equivalent, zero wave number. Other signals having other velocities and wave numbers will be attenuated as indicated by the pattern. By the introduction of delays into seismometer outputs before summing the array can be steered to pass unattenuated plane waves arriving from any specified azimuth having any specified horizontal phase velocity. At any given frequency this processing has a gain characteristic in wave number space with exactly the same shape as the straight sum pattern but which is shifted to have its peak at the location of the signal at that frequency. Thus the straight sum pattern is a useful measure of the selectivity of any array even when the array is steered to a signal having other than infinite phase velocity.

III.   PATTERN REQUIREMENTS AND APPROACHES TO SYNTHESIS

The acceptability of patterns depends greatly upon the noise which is to be rejected when the array is steered to an event of interest and upon the expected location in frequency wave number space of the interesting events. Figure 4 is a map of the world as seen by an array centered on Miles City, Montana, when only the initial P-phase signal at one Hertz is considered.[11] Since the initial P-phase arrival usually has most frequency content at about one Hertz that map is a good picture of the location of teleseismic signals of interest and of noise which results from distant seismic activity and reaches the array via earth paths similar to those of interesting signals. From the consideration of such a picture of the earth as seen in a slice through frequency wave number space it has been decided that any LASA which is to be used for the detection of underground nuclear tests should have a main beam with a radius of about .005 cycles/km or less in wave number space. That is, if the gain of the LASA is measured at a distance of more than .005 cycles/km from the point in wave number space at which it is steered, then the gain must be significantly lower than unity. This appears to allow for a sufficient degree of selectivity in steering the beam to some point while not passing similar signals from other parts of the globe.

If subarray or array geometries become very regular, one must consider a phenomenon known as aliasing. If a high frequency time function is sampled at too low a rate, the sampled waveform can appear exactly the same as the sampled waveform for a much lower frequency signal.[2] Similar problems arise when spatial signals are sampled on a regular grid.[3] Then the confusion is in wave number. For example,

Figure 17B is a section of the pattern of the regular geometry shown in Fig. 15B. Periodic signals coming from the east with wave numbers greater than .04 cycles/km cannot be distinguished, after spatial sampling by the array, from some other signals having wave numbers in the range $0 - .04$ cycles/km. There is a bad sidelobe at .08 cycles/km, exactly twice the wave number at which aliasing first becomes a problem. Similar lobes appear at all multiples of .08 cycles/km in wave number space.

Even small changes in the spatial sampling positions can eliminate true aliasing as a real problem. However, severe sidelobes, almost as large as unity, can still exist in regions too close to the main beam of the pattern. In this report attention has been focussed upon such lobes, known as grating lobes, rather than upon aliasing. This was done since it is the grating lobes which can cause trouble, not aliasing. when arrays are not extremely regular.

Consideration of possible and expected noise sources has led to the conclusion that a full array pattern should have no grating lobes closer than one to two cycles/km to the center of the main beam in wave number space.[6,9] The problems of aliasing are avoided by assuming a sufficiently irregular placement of subarrays and of seismometers within a subarray.

A variety of approaches to the problem of synthesizing satisfactory patterns using about 500 seismometers do exist. They all have in common the characteristic that they all use some kind of trial and error method combined with reasonable intuition. One approach, the one which we have used most extensively, is to restrict the arrays to be compound arrays similar to the experimental LASA. The full array pattern can

be synthesized by almost independent synthesis of the array and subarray patterns. In particular, the array pattern might be synthesized using 21 points to yield a .005 cycles per km half beamwidth or less, and to have no bad sidelobes closer than .1 cycles/km to the center of the main beam. This choice of .1 cycles/km appears to give a reasonable tradeoff between requirements upon the array and the subarrays. The subarray can then be designed to have a main beam radius of .1 cycles/km or less and no bad sidelobes closer than one to two cycles/km. Such a compound array would then meet the design requirements. The design of array or subarray patterns using from 20 to 25 points must proceed via trial and evaluation methods. Relatively algorithmic improvement methods such as are presented in Section VII can at best be of value to find perturbations of a geometry which would yield an improved pattern. One is stymied when no nearby geometry will yield a better pattern but there may be a very different geometry having a much more satisfactory pattern. Tradeoffs between the requirements imposed upon the array and subarray patterns are discussed in Section VI.

One might attempt full array pattern synthesis using some of the formalized trial and error methods which have been developed for the synthesis of radar and sonar antenna patterns.[1] The synthesis of a LASA geometry is very similar to the synthesis of an extremely thinned array. However, given the number of seismometers, the width of the main beam, and constraints upon the location of the first grating lobes, the requirements of the LASA are very much different from those of radar arrays. In the radar case sidelobe levels less than 20 db down from the main beam would be considered excessive. Such low sidelobe levels can be maintained for thinned arrays only by

increasing the width of the main beam. For the LASA such an increase in main beam width is not allowed. Neither is a significant increase in the number of seismometers. The LASA can then satisfy the design criteria only by a significant increase in sidelobe level. It is thus our opinion that application of the methods suggested for density tapering of radar arrays cannot yield sidelobe levels significantly lower than those which have been achieved using compound arrays. Since direct synthesis using all of the seismometers would not constrain the array to be a compound array, such an approach has not been pursued. The advantages of compound arrays (see Section IV) are sufficiently great to reject a direct synthesis of a full array since the direct synthesis does not promise any significant reduction of sidelobe level in the region of wave number space of interest.

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IV. COMPOUND ARRAYS: FAMILIES OF ARRAY GEOMETRIES

Many engineering advantages are gained by the use of compound arrays.
Considerations related to the design of LASA electronics modules, communication
channels, and power distribution systems all argue strongly in favor of the use of com-
pound arrays. The use of such arrays also provides natural divisions for data pro-
cessing schemes. Individual subarrays can be processed and then the results combined
in a second stage of processing. Failures in a single subarray need not effect the re-
duction of data from another subarray.

For convenience in establishing power systems and communication channels it
is desirable that both array and subarrays be set out along several radial arms connected
to a central point. Such a configuration tends to keep the length of lines or number of
separate communication channels within bounds. The use of configurations which
complicate the power distribution or communication networks should be used only if
considerable increase in the capabilities of the array can be gained. No such geome-
tries have yet been discovered.

All other considerations being equal, arrays graded to be more dense towards
the center are desired over others. This grading is a hedge against possible loss of
coherence of teleseismic P-phase arrivals as the signal propagates across the array.

Array geometries having the most remote subarrays on a circle of radius
100 km have been studied. Such a dimension was picked since an array of approximately
that size must be used in order to obtain the narrow main beam which is required. An

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array should have at least three subarrays out at the 100 km radius circle. More than the minumum three are desirable to allow the use of redundancy in the determination of epicenters using a limited network composed of a single output from each of the sub-arrays on the 100 km circle.

Subject to the above constraints a relatively extensive search for possible suitable locations of subarrays has been completed. Figures 5 through 8 show typical members of the families investigated as possible deployments for 21 subarrays. The number of points on a ring and the number of rings is fixed for each family. The spacing of rings is geometric with the ratio of the radius of adjacent rings being a constant. This constant has been varied to generate the members of each family. The same configurations, but with arithmetic spacing of rings, have also been evaluated. A few arrangements with subarrays on spiral rather than straight arms and with random placement of subarrays were considered and discarded on the basis of their patterns. Except for a few trials the number of subarrays has been maintained at 21, the number in the experimental LASA which has been installed in Montana. This appears to allow a reasonable tradeoff between the number of subarrays and the number of seismometers per subarray. There is no reason at the present time to believe that another number of subarrays, keeping the total number of seismometers constant, could yield significantly better performance by a LASA.

A large amount of effort has been expended to obtain subarray geometries as opposed to array geometries. The question of subarray geometry is discussed in Section VI. Briefly, one of the array geometries, suitably scaled, could be used for a

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subarray with 21 seismometers, or other subarray geometries using a different number of seismometers could be employed.

V.   BEST ARRAY GEOMETRIES AND THEIR PATTERNS

Figures 9 through 14 show the geometries and associated patterns of the most promising arrays found in each of the families of arrays described in the preceding section. The families with five or seven subarrays on each ring appear to be the most desi   le.

Array patterns have been graphically presented in two forms. The pattern is a gain function defined on the two-dimensional space of horizontal wave number. Such patterns have been graphically presented in figures such as 9B as contour plots. The lines on the plot represent the level of the pattern. These plots were obtained by smoothly connecting the points on a plot printed by the line printer attached to a large scale digital computer. A listing of a Fortran II program which can be used to generate the printer plots and a description of its use and operation are contained in Appendix A.

Plots such as that shown in Figure 9C were obtained by evaluating the pattern along a ray leaving the origin of wave number space. The plots are thus cross sections through the array patterns. Restriction to only a few radial directions and the use of a more accurate output device than a printer plot allowed a more accurate and detailed picture of the pattern to be obtained in directions of particular interest. Plots were obtained in those directions in which the cruder printer plots indicated the most severe sidelobes v   d be found. A listing of Fortran IV programs used to obtain cross sec-tional plots is given in Appendix B. That appendix also describes the operation and use of those programs. The programs have been constructed so that minimal modifications

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would allow one to obtain cross sectional patterns without the use of a Stromberg Carlson 4020 plotter which was the output device used.

The programs described above can also be easily modified to help interpret in wave number or frequency wave number space the effect of processing schemes much more complex than the straight sum processing used for the evaluation of alternative geometries. As mentioned in Section II the interpretation of more complex processing in frequency wave number space is outside the scope of this report and has not been included.

In each family the arrays were judged most satisfactory when they had the least severe sidelobes in the range from zero to .1 cycles/km and had beam widths which were sufficiently small. As discussed above the beam width should be .01 cycles/km or less. Sidelobes further than .1 cycles/km from the main beam will be sufficiently attenuated by subarrays if the subarrays are properly set out. Since no single scalar quantity can adequately represent the quality of the various arrays, the choice of best was necessarily somewhat subjective. At least one representative best member from each major family investigated has been saved.

Comparisons between members of the same family or of different families of array geometries is made a bit less complex by noting certain symmetries in patterns and invariances with geometries. Note that contour maps of patterns have been drawn only over relatively restricted pie shaped sections of wave number space. Only those sections are necessary since the entire pattern is obtainable simply by completing the

circle with identical pie slices. This result and certain invariances for patterns of arrays with odd numbers of subarrays per ring are described in more detail in Appendix C.

The array configuration shown in Figure 13A, which has been picked as the best geometry with four subarrays per ring, yields a pattern which appears to be definitely inferior to those which use three, five or seven subarrays per ring. The pattern for the geometry shown in Figure 13A has significant sidelobes which are no further than .015 cycles/km away from the center of the main beam. Even more severe sidelobes appear at .05 and .07 cycles/km from the main beam. This best member of the family of arrays with four subarrays per ring is quite similar to the actual configuration of the experimental LASA which has been installed in Montana. Similar severe sidelobes have been noted for that experimental array.[4,7,11] These can be seen in Figure 2.

It is our opinion that the array shown in Figure 14A which has three subarrays per ring can also be rejected as the most valuable array. The pattern for this array has a broader main beam than that of other geometries under consideration. Sidelobes are not excessive, but are slightly larger in amplitude than those of alternative patterns. If the array is stretched over a larger area to reduce the width of the main beam, then the sidelobes will move in closer. Finally, more than three subarrays are desirable on the largest ring for the purpose of determining epicenters using the array.

The configuration shown in Figure 10A is probably to be preferred to that of Figure 9A which also has seven subarrays per ring. The array in Figure 9A has

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subarrays concentrated more towards the 100 km radius ring. Since the validity of patterns depends somewhat upon the coherence of a signal across the array, this more spread out array would probably be more adversely affected by loss of coherence. Indeed, any signal processing scheme which depends upon signal coherence will be more seriously affected by loss of coherence when subarrays are spread out than when they are clustered together. One might consider scaling down the size of the array shown in Fig. 9A since it appears to have a very narrow main beam. This should be avoided since it will seriously effect the usefulness of the outer ring of the array for the determination of epicenters. The decision between these two arrays is at best subjective.

A comparison of the geometries of Figures 11A and 12A, having five subarrays per ring, with the configurations having seven per ring is completed in Section VII. That comparison is deferred because the application of a gradient technique described there has resulted in some changes in the sidelobes of the pentagonal arrays.

VI.   TRADEOFF BETWEEN SUBARRAYS AND ARRAY IN A COMPOUND ARRAY

In the above sections it has been rather casually assumed that any bad sidelobes of the array pattern farther out than .1 cycles/km could be sufficiently attenuated by the subarrays to be negligible. It has even been implicitly assumed that some attenuation, say a few db, might be obtained at wave numbers having magnitude as small as .05 cycles/km or less. These assumptions are discussed in this section and the importance of the relative sizes of array and subarrays is emphasized.

Assume that the array geometry shown in Fig. 10A is used and that each subarray has seismometers in the same configuration but that the largest ring of seven seismometers has a 10 km diameter. The subarray is then 1/20 the size of the array. The gain of such a compound array is found by forming the product of the array and subarray gains.

Consider the most severe array sidelobe shown in the center frame of Fig. 10C. That lobe is about .037 cycles/km from the origin and has level .42. The subarray pattern will have the same shape but the wave number scale will be multiplied by the ratio of array to subarray diameters. If the subarray has diameter 10 km, it will be able to reduce the above mentioned array lobe to a level of about .36. The use of a subarray having a diameter of 15 km would attenuate the lobe even more to about .3. If the subarray diameter were reduced to 7 km, this same lobe would remain larger than about .38. These figures are approximate only because subarray gains have been obtained from Fig. 10C by changing the wave number scale. The point is that even for wave numbers as small as .037 cycles/km some meaningful attenuation can be obtained

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from subs   ys having diameters from 10 to 15 km. The attenuation from a 7 km
subarray is very slight so close to the origin of wave number space.

The larger subarray diameters maintain similar advantages all the way out to
.1 cycles/km. The array sidelobes are more effectively attenuated by subarrays with
diameters 10 to 15 km than by those with diameter 7 km. This will tend to be true
independently of the particular subarray configuration which is used.

If the configuration of Fig. 10A, scaled to have a diameter between 10 and
15 km, is used as a subarray, then the .42 level closest sidelobe will appear between
.48 and .74 cycles/km from the center of the subarray pattern. This sidelobe is not
extremely detrimental and may be tolerable in the subarray pattern.

Figure 17B is a cross section of the pattern generated by the geometry of a
subarray of the experimental LASA. The configuration has been scaled to a 200 diam-
eter circle for convenient comparison. The true subarray lies w. .in a circle with a
diameter of 7 km. Figure 15B shows the seismometer configuration. Observe that the
pattern for this configuration has a close in sidelobe at level .44, higher than the one
discussed above for a subarray using only 21 elements. Even if the subarray has
diameter 7 km, this lobe is at .4 cycles/km. In general, the pattern drawn in Fig.
10C is definitely superior to that drawn in Fig. 17B. The superiority of the 21-element
subarray in terms of sidelobes is maintained even when its size is increased to 10-15
km while the array using 25 seismometers is maintained at 7 km. The 21-element sub-
array has a generally lower sidelobe level and has it over a larger range of wave num-
bers than the 25 element subarray does.

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It seems that the use of 21 seismometers in a subarray, placed on three rings, containing seven seismometers, may yield a subarray with characteristics superior to those of the LASA Montana subarrays which use 25 seismometers. One might then expect that a subarray configuration which uses 25 seismometers and is definitely superior to either of those discussed above could be found. No extensive search for such a geometry has been completed. However, one family of geometries, having five rings spaced geometrically and each having five seismometers per ring, has been briefly investigated. Figure 15A shows the resulting subarray and Figs. 16 and 17A show patterns. The lobe of level .49 at a distance of .05 cycles/km from the center of the main beam is the worst characteristic of the pattern. This lobe might be removed using sensitivity functions as described in the next section or a completely different family using 25 seismometers might be investigated. The synthesis of configuration using 25 points has not been pursued.

VII.   SENSITIVITY AND PATTERN IMPROVEMENT

The sensitivity function of a pattern or array is a measure of how much the pattern will be effected by small changes in seismometer or subarray positions. There are two principal reasons for obtaining a sensitivity function. First, of course, such a function indicates how accurately subarrays must be located so that the pattern will not be seriously modified by errors in placement. Such a function can also be used to help find improved geometries. It can indicate how the geometry should be perturbed to achieve certain desired pattern changes. A useful sensitivity measure is defined in the following paragraphs.

Let $\bar{r}_q$ be the two-dimensional vector which locates the qth subarray in an array or q^{th} seismometer in a subarray. Suppose that $\bar{u}_q$ is a unit vector which indicates the direction in which the qth element might be moved and that $F(\bar{r}_q)$ is a complex valued function of $\bar{r}_q$. The function F can of course be a pattern with dependence upon the position of the qth point of the array made explicit. If it exists, the following directional derivative can be defined.

$$\frac{\partial F}{\partial \bar{u}_q} = \lim_{\epsilon \downarrow 0} \frac{F(\bar{r}_q + \epsilon \bar{u}_q) - F(\bar{r}_q)}{\epsilon} = \frac{\partial F(\bar{r}_q + \epsilon \bar{u}_q)}{\partial \epsilon} \bigg|_{\epsilon = 0}$$

It is a matter of straightforward substitution to verify that if

$$F = F_1 F_2$$

then

$$\frac{\partial F}{\partial \overline{u}_q} = F_1 \frac{\partial F_2}{\partial \overline{u}_q} + F_2 \frac{\partial F_1}{\partial \overline{u}_q}$$

which is the usual rule of differentiation of products. Other rules such as

$$\frac{\partial F^s}{\partial \overline{u}_q} = s F^{s-1} \frac{\partial F}{\partial \overline{u}_q}$$

also hold for the directional derivative.

A physical meaning can easily be attached to $\partial F/\partial \overline{u}_q$. To a first order approximation we have

$$F(\overline{r}_q + \epsilon \overline{u}_q) = F(\overline{r}_q) + \epsilon \frac{\partial F(\overline{r}_q)}{\partial \overline{u}_q}$$

if ϵ is sufficiently small. If F is a pattern, then $\epsilon [\partial F(\overline{r}_q)/\partial \overline{u}_q]$ is roughly the change in the pattern when subarray or seismometer q is moved ϵ km in the direction of $\overline{u}_q$.

Let $\langle \overline{k}, \overline{u}_q \rangle$ and $\langle \overline{k}, \overline{r}_q \rangle$ denote usual scalar products of the indicated vectors. Then the magnitude of

$$H = \frac{1}{N} \sum_{j=1}^{N} \exp[-i 2\pi \langle \overline{k}, \overline{r}_j \rangle] \ ,$$

which equals $(HH^*)^{1/2}$, is the pattern of an array. One can easily obtain

$$\frac{\partial H}{\partial \bar{u}_q} = -i \frac{2\pi}{N} \langle \bar{k}, \bar{u}_q \rangle \ \exp\left[-i\,2\pi \langle \bar{k}, \bar{r}_q \rangle\right] . .$$

From this

$$\frac{\partial HH^*}{\partial \bar{u}_q} = -\frac{4\pi}{N^2} \langle \bar{k}, \bar{u}_q \rangle \ \mathrm{Im}\left\{ H \exp\left[i\,2\pi \langle \bar{k}, \bar{r}_q \rangle\right]\right\}$$

and

$$\frac{\partial |H|}{\partial \bar{u}_q} = \frac{\partial (HH^*)^{1/2}}{\partial \bar{u}_q} = -\frac{2\pi}{N|H|} \langle \bar{k}, \bar{u}_q \rangle \ \mathrm{Im}\left\{ H \exp\left[i\,2\pi \langle \bar{k}, \bar{r}_q \rangle\right]\right\} .$$

Considered as a function of $\bar{k}$ this is a sensitivity function which reflects the effects of moving the qth point on an array in the $\bar{u}_q$ direction.

It is convenient and useful to express $\frac{\partial |H|}{\partial \bar{u}_q}$ in terms of polar coordinates of all quantities. Let $|H|$ and φ be the magnitude and phase of the complex gain H. Let θ_q be the angle of $\bar{u}_q$ and let r_q, ψ_q be the magnitude and direction of the position $\bar{r}_q$. Finally let k and θ be magnitude and direction of $\bar{k}$. Using this notation, $\frac{\partial |H|}{\partial \bar{u}_q}$ can be expressed as

$$\frac{\partial |H|}{\partial u_q} = -\frac{2\pi}{N} k \ \cos (\theta - \theta_q) \sin \left[\varphi + 2\pi k r_q \cos (\theta - \psi_q)\right] .$$

It is this expression using polar notation which has been most useful.

It is now very easy to obtain an upper bound upon the possible effects of moving a seismometer or subarray. In particular,

$$\left| \frac{\partial |F|}{\partial \bar{u}_q} \right| \leq \frac{2\pi k}{N} \quad .$$

This bound on sensitivity is inversely proportional to the number of elements and
directly proportional to the size of the wave number at the point in wave number space
which is of interest. Small changes in subarray positions can greatly effect the pattern
for large values of wave numbers. Note also that this bound is independent of r_q, the
magnitude of the position vector for the qth element. Roughly speaking, a change of
ε km in the position of an array point cannot change the pattern in wave number space
more than $2\pi k \varepsilon / N$. Thus a change of 1 km in the position of one element of a 21-element
array of subarrays cannot change the pattern by more than $(.3)$ k. For k less than
.1 cycles/km, this is less than .030. Based upon this bound one can conclude that
changes as large as 1 km in the location of subarrays may not result in drastic
changes in patterns. Certainly the pattern of a modified array must be checked but it
is hoped that changes as large as one kilometer in position can be tolerated without un-
due concern. Similar computations for subarrays covering a region having a 10 km
diameter indicate that errors on the order of .05 km may not have too adverse effects
although the effects must be checked. Of course, if these bounds are divided by the
number of elements, then the check is not needed as long as changes as large as .03
in pattern are considered acceptable. Such an approach requires much more accurate
placement of elements.

It may not be physically possible to locate a subarray less than a kilometer away
from a point which is specified for an idealized array. Such a situation need cause no

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serious difficulty. Suppose some realizable alternative to an idealized array configuration is under consideration. The pattern for the configuration can be quite completely obtained in the regions of wave number space of interest using programs such as are listed in the appendices. A study including two contour plots and a dozen cross sectional plots would require only about five minutes of time on an IBM 7094 computer. The cost of such a study of an array configuration is so slight that it can always be done for any configuration under serious consideration.

Since it is so inexpensive to evaluate a pattern, it seems reasonable to place bounds on allowed variations in subarray locations by specifying a surface, defined in wave number space, below which the array pattern must lie. Such a requirement could be used in conjunction with quite weak requirements upon allowed changes in subarray position from idealized positions. For example, changes of position of the order of 5-10 km would not be unacceptable if the pattern was not deteriorated by those changes. If such a method of specification is used, the strong requirements on position imposed by the use of sensitivity functions can then be relaxed.

Some success has been achieved in using sensitivity functions to improve patterns by moving subarray locations. A Fortran IV program has been written which evaluates sensitivity of patterns with respect to the radius of a ring of elements in an array. This is done simply by setting θ_q equal to ψ_q and summing over those q which index the elements of the ring of interest. The program computes and plots this sensitivity function for points along a ray in wave number space. The program is listed and its use described in Appendix D.

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Figure 18 shows the sensitivity of a pattern to changes in the radii of the five

element rings of the array shown in Fig. 12A. The sensitivity has been evaluated along

a direction in wave number space in which are found the most severe sidelobes of the

pattern. These sensitivity functions, as well as all others plotted in this report,

should be divided by the number of elements (21) to agree with sensitivity as defined

above.

It was determined using the sensitivity function, that the bad lobe at about

.0425 cycles/km in wave number could be reduced by reducing the radius of the ring of

elements having a 25 km radius. Judging from the sensitivity, this change should re-

duce the lobe at .07 cycles/km a comparable amount if not more.

Figure 19 shows the effect upon pattern and sensitivity of reducing the inner

ring radius to 20 km. The plots are along the same direction in wave number space as

those of Figure 18. The lobe at .0425 cycles/km has been reduced from .52 to .43,

a nontrivial improvement. The second severe lobe did not fare so well. As the radius

of the ring decreased, the sensitivity changed in such a way that the net effect upon the

lobe was slight.

Figure 20 shows the effects upon the pattern in a different direction in wave

number space of the changes described above. The direction was that having the most

severe adverse effects. Specifically, the lobe at about .055 cycles/km has grown to a

value of .56. This degradation was roughly predicted by the sensitivity function eval-

uated along the direction 18° north of east.

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The relative quality of the patterns of the modified and unmodified arrays is not clearly defined. It is our opinion that the reduction of the level of the nearest sidelobe looking eastward is worth the cost paid in other directions. In general the adverse effects of sidelobes becomes greater as they get closer to the main lobe. Very close sidelobes cannot be killed by subarray processing.

No other modifications of this basic geometry have been attempted. Sensitivity functions have not indicated any changes expected to yield significant pattern improvement.

A procedure similar to that described above has also been followed starting with the array shown in Fig. 11A. The results are summarized in Figs. 21, 22, and 23. By increasing the ring having radius 55 km to 57 km the lobe at about .063 cycles per kilometer in the eastward direction has been reduced from .5 to .43. The lobe at about .054 has increased from .35 to .44. The relative quality of the modified and unmodified arrays is so tenuous that no attempt is made to rank them.

No further attempts at modifying these patterns have been made. It is possible, for example, by decreasing the radius of the 30.25 km ring, that marginal improvement in sidelobes could be obtained. The small amounts of expected improvement did not seem significant enough to warrant any further search.

Figures 24 and 25 are pattern and sensitivity plots for the arrays of Figs. 9A and 10A, respectively. The directions in wave number space were picked to exhibit most severe sidelobe activity. No changes of radii of rings appear to promise improved sidelobes.

After considering all the various patterns described in this report, it is our opinion that the geometry of Fig. 10A using seven subarrays per ring is the best to use for a LASA Unfortunately, it does not appear that the lobe of level .42 at a distance of .037 from the main beam can be removed by slight changing of the radii of rings. Even with that lobe this geometry seems most desirable. No other bad lobes are present. This is not true of geometries having five subarrays per ring. Those arrays have first significant lobes a bit farther out but have a generally higher sidelobe level and have other very significant sidelobes closer than .1 cycles/km from the center of the main beam.

VIII.   A COMPARISON WITH THE EXPERIMENTAL LASA

Figures 2 and 10C show cross sections of the patterns for the experimental LASA geometry and for a proposed geometry which would deploy 21 subarrays on three rings. For wave numbers in the range .02 – .2, the sidelobe levels are roughly the same for the two geometries. However, for wave numbers less than .02 in magnitude the proposed configuration shown in Fig. 10C yields a pattern very much better than the pattern of the experimental LASA.

The experimental LASA, depending upon one's point of view, either has a main beam nearly .04 cycles/km in diameter or has a bad sidelobe only .015 cycles/km from the center of the main beam. The proposed configuration has no such close sidelobe and has a main beam with a diameter of about .01 cycles/km in wave number space. The pattern for the proposed geometry shown in Fig. 10A is sufficiently better than the pattern for the experimental LASA to warrant serious consideration if any other LASAs are constructed. It should be noted that pattern improvement has been obtained primarily in the important region of wave number space close to and including the main beam.

IX. SUMMARY

A study of possible geometries for a Large Aperture Seismic Array (LASA) has been completed. For a variety of reasons the array was restricted to being composed of 21 subarrays deployed over a circular region having a diameter of 200 km. From each of several families of array geometries the members yielding best patterns were saved and discussed. Two families, those having three or four subarrays on each of several rings, were discarded as definitely inferior to other families. The relative quality of arrays using five or seven subarrays per ring was not as clear. The geometry shown in Figure 10A appears to be the best all around compromise. That geometry yields a significantly better main beam than the experimental LASA does. Other possible geometries and their patterns have been saved for comparison.

The diameter of subarrays in the experimental LASA in Montana is 7 km. Pattern studies indicate that this should be increased to from 10 to 15 km for other installations. Even using from 20 to 25 seismometers per subarray, undesirable aliasing and unwanted grating lobes can be avoided by the use of suitable subarray geometries.

Sensitivity functions for array patterns have been developed. These indicate that position changes of the order of .5% of the diameter of an array or less should have only slight effect upon the array pattern. Fortunately it should not be necessary to maintain such accurate placement of subarrays with respect to some idealized array. The cost of evaluating the pattern of a configuration is small enough to allow this to be

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done for any configuration under serious consideration. Any changes in a configuration can be directly evaluated in terms of the pattern. The use of sensitivity functions to improve geometries has been investigated with moderate success.

PROGRAM-FOURIER TRANSFORM WITH PRINTER PLOT

The Fortran II program listed below computes the pattern defined by (1) for values of k_x, k_y on a rectangular grid (100 × 100) in the wave number plane. Execution of the program generates a printer plot which is a map of the pattern. If instruction 111 and the one preceding it are removed from the program, then the program can be used to generate gain plots in wave number space resulting from weighted sum processing rather than straight sum processing. The inclusion of these instructions avoids the need to punch a one on each card which locates a subarray.

The Plot

The plot is a contour map over a rectangular region of k_x, k_y space. One corner of the rectangle is at position (XK, YK). The diagonally opposite corner is at (XK + RX, YK + RY). These points are at the upper left and lower right corners of the output plot.

The user specifies a parameter DELTA where $0.05 < \text{DELTA} < 1$. The range of the magnitude of gains is divided into intervals $[(h-1)\,\text{DELTA}, h\text{DELTA}]$ with $h = 1, 2, \ldots$ Those points (k_x, k_y) on the grid for which

$$(h-1)\,\text{DELTA} \leq |\text{Gain}| < h\text{DELTA}$$

are marked with the hth character from the list

$(1, 2, 3, 4, 5, 6, 7, 8. 9, A, B, C, D, E, F, G, H, I, J, K, L, M, N, \text{blank})$.

In order to aid the user to read the contour plot, blanks are substituted for characters when several contiguous characters are the same. Only the left most of such a group of characters is written in any row.

Use of the Program (Fortran II)

Following the *DATA card, there must be four cards containing octal coding, format (6012), for the characters in the list given above. Following these four cards there can be any number of sets of data. Each set of data contains a card with format (2I5, 5F1∅. 5) containing N, NCTR1, DELTA, RX, RY, XK, YK. The variable N specifies the number of seismometers. NCTR1 is a control number. If NCTR1 = 3, we obtain a plot of the magnitude of gain. If NCTR1 = 2, then a listing of N times the gain is given. If NCTR1 = 1, both the plot and the listing are given. The variables DELTA, RX, RY, XK, YK have been described above. Within each set of data, this card is followed by N cards with format (42X, 3F1∅. 3). They contain (y_j, x_j, w_j), $j = 1, \ldots, N$ where y_j, x_j are the north and east components of the location of the jth seismometer or subarray and w_j is the weight attached to it for weighted sum processing. For pattern studies the w_j need not be punched and are set to unity by instruction 111.

The four octal coded cards appear only once, immediately after *DATA, regardless of the number of sets of data processed.

The computation of a plot using 21 seismometers requires about one minute or less of time on an IBM 7094 computer.

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```
CFTWP      FOURIER TRANSFORM WITH PRINTER PLOT------LACOSS 9/22/65
C          MODIFIED NOV 8,1965----- DOES NOT REQUIRE READING OF WEIGHTS
C          ASSUMES ALL WEIGHTS TO BE UNITY
           DIMEN' ON X(600),Y(600),SNTBL(2401),CSTBL(2401),VALU(600)
           DIMENSION RNUM(600),PRINT(100),ALPHA(24)
           DIMENSION W(600)
           READ INPUT TAPE 2,2004,(ALPHA(J),J=1,24)
      2004 FORMAT(6012)
C
C          FORM SIN AND COS TABLES
           DO 10 I=1,1200
           FLI=I
           THETA=(3.14159265/1200.)*(FLI-1.)
       10  SNTBL(I)=SINF(THETA)
           DO 12 I=1201,2401
       12  SNTBL(I)=-SNTBL(I-1200)
           DO 14 I=1,1801
       14  CSTBL(I)=SNTBL(I+600)
           DO 16 I=1802,2401
       16  CSTBL(I)=SNTBL(I-1800)
C
      999  REWIND 8
           READ INPUT TAPE 2,1000,N,NCTR1,DELTA,RANGEX,RANGEY,XKST,YKST
      1000 FORMAT(2I5,5F10.5)




           WRITE OUTPUT TAPE 3,3000,N,DELTA,RANGEX,RANGEY,XKST,YKST
      3000 FORMAT(42HVALUES OF N,DELTA,RANGEX,RANGEY,XKST,YKST/1I10,3F15.5)
C          N--NUMBER OF SEIS
C          DELTA-AMT BETWEENLEVELS
C          RANGEX,RANGEY--EXTENT OF K-PLANE PLOT IN CYCLES PER POSITION UNIT
C          XKST,YKST--COORD OF PT FROM WHICH RANGEX,RANGEY MEASURED
           READ INPUT TAPE 2,1001,(Y(K),X(K),W(K),K=1,N)
      1001 FORMAT(42X,3F10.3)
           DO 111 K=1,N
      111  W(K)=1.
           WRITE OUTPUT TAPE 3,3001,(Y(K),X(K),W(K),K=1,N)
      3001 FORMAT(16HVALUES OF Y,X,W/(3F20.5))
           SNORM=0.0
           DO 100 I=1,N
      100  SNORM=SNORM+W(I)
           IF(SNORM)117,112,115
      112  CALL EXIT
      115  SNORM=ABSF(SNORM)
           GO TO 118
      117  SNORM=-ABSF(SNORM)
      118  CONTINUE
           DX=RANGEX/99.0
           DY=RANGEY/99.0
C
           DO 22 M1=1,100
           DO 21 M2=1,100
           SUMR=0.0
           SUMI=0.0
           FM1=M1
           FM2=M2
           XK=XKST+(FM2-1.)*DX
           YK=YKST+(FM1-1.)*DY
           DO 20 J=1,N
           ARG=-XK*X(J)-YK*Y(J)
           ARG=MODF(ARG,1.)
           IF(ARG)160,165,165
      160  ARG=ARG+1.0
      165  ARG=2400.*ARG
           NARG=ARG
           K=NARG+1
           KK=K+1
           RATIO=MODF(ARG,1.)
           COSVAL=CSTBL(K)+RATIO*(CSTBL(KK)-CSTBL(K))
           SINVAL=SNTBL(K)+RATIO*(SNTBL(KK)-SNTBL(K))
           SUMR=SUMR+W(J)*COSVAL
       20  SUMI=SUMI+W(J)*SINVAL
           VAL=SQRTF((SUMR**2)+(SUMI**2))
       21  VALU(M2)=VAL
```

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```
       WRITE TAPE 8,(VALU(K),K=1,100)
       GO TO (45,47,48),NCTR1
   45  WRITE OUTPUT TAPE 3,1002,YK
 1002  FORMAT(4H YK=F20.5)
       WRITE OUTPUT TAPE 3,1003,(VALU(K),K=1,100)
 1003  FORMAT(1H ,10F12.5)
       PLOT=1.0
       GO TO 50
   47  WRITE OUTPUT TAPE 3,1002,YK
       WRITE OUTPUT TAPE 3,1003,(VALU(K),K=1,100)
       PLOT=0.0




       GO TO 50
   48  PLOT=1.0
   50  CONTINUE
   22  CONTINUE
       REWIND 8
       IF(PLOT) 52,   51,52
   51  GO TO 999
   52  IF(DELTA-0.05) 53,54,54
   53  WRITE OUTPUT TAPE 3,2000,DELTA
 2000  FORMAT(1H ,22HDELTA TO SMALL(DELTA=  ,F10.3 )
       GO TO 999
   54  WRITE OUTPUT TAPE 3,2001,DELTA
 2001  FORMAT(1H1,15HINCREMENTS OF      ,F10.3)
       WRITE OUTPUT TAPE 3,2002
 2002  FORMAT(1H ,10HLINEHEADER,100H1234567890123456789012345678901234567
      18901234567890123456789012345678901234567890123456789012345678901234567890  )
C           PAGE HEADER
       KFST= (1.0 /DELTA) - 0.05
       DO 69   LINE =1,100
       READ TAPE 8,(VALU(K),K=1,100)
       DO 55 K=1,100
   55  VALU(K)=ABSF(VALU(K))/SNORM
       KSTEP =KFST
       DO  68 J=1,100
       RNUM(J)=VALU(J)
       IDIOT=1
   62  STEP =KSTEP
       CLEV1 = STEP*DELTA
       CLEV2 = CLEV1 + DELTA
       GO TO(625,625,64),IDIOT
  625  IF(RNUM(J)-CLEV1)63,64,64
   63  KSTEP = KSTEP - 1
       IDIOT=2
       CHNG = 1.0
       GO TO  62
   64  GO TO (645,66,645),IDIOT
  645  IF(RNUM(J)-CLEV2)66,65,65
   65  KSTEP =KSTEP+1
       IDIOT=3
       CHNG = 1.0
       GO TO 62
   66  IF (J -1)  67,72,67
   67  IF (CHNG -1.0)   71,70,71
   71  PRINT(J)=ALPHA(24)
       GO TO  68
   70  NCHAR =KSTEP +1
       PRINT(J) = ALP(NCHAR)
       CHNG =0.0
       GO TO 68
   72  KFST= KSTEP
       GO TO  70
   68  CONTINUE
       LIDX=LINE
  800  IF(LIDX-10) 802,801,801
  801  LIDX=LIDX-10
       GO TO 800
  802  CONTINUE
       WRITE OUTPUT TAPE 3,2003,LIDX,(  PRINT(J),J=1,100),LIDX)
 2003  FORMAT(1H ,I4,6H          ,100A1,6H      ,I4)




   69  CONTINUE
       GO TO 999
       END
```

APPENDIX B

PROGRAMS-EVALUATION OF GAIN IN FREQUENCY WAVE NUMBER SPACE

The programs described and listed in this appendix were used to generate cross sections of patterns. If instruction 39 and the one GO TO 39 preceding it in the program 3DFT are removed, the same program and subroutines can generate cross sections of gain at any specified frequencies for any linear filter and sum processing scheme.

Form of Data Output

The 3DFT program and associated subroutines compute complex gains of a processing scheme along a ray in wave number space for a fixed frequency. The gain is evaluated at increments of DELTAK in wave number from zero to NK*DELTAK. For each of NFREQ values of frequency, FREQ(I) = 1,...,NFREQ, the gain is computed along NTHETA rays in wave number space. THETA(I), I = 1,...,NTHETA. The gain along each ray at each frequency can be output in several forms. The choice of points to evaluate gains and the choice of output format are made in subroutines UTIL and/or OUTPU. Thus, these can be modified without destroying the entire program.

In the current form of OUTPU several options are available once the complex gain has been computed along a ray in wave number at a fixed frequency. The gain magnitude and phase can be plotted using a Stromberg Carlson 4020 plotter. The log of the magnitude of gain and the phase can be plotted. The gain or log of the gain can be printed out in addition to being plotted. These possible outputs correspond to a control variable, NCTRL taking the values 1, 2, 3, or 4, respectively. Two variables,

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STO1U and STO1L, set upper and lower levels at which gain magnitude or log of the gain magnitude are clipped before a plot is made. The routines KWKPLT, STO1D, CAMRAV, and PLTND used in the programs are routines for plotting using the Stromberg Carlson 4020.

Use of the Program (Fortran IV)

The first data card contains NRUNS in Format (I5). That number determines how many sets of data are to be processed. Then follow NRUNS sets of data. Each set is composed as described below.

The first card of a set of data contains NS, NFP, TFIRST, DELTAT, (2I5, 2F1Ø.5). NS is the number of seismometers or subarrays. NFP is the number of time samples in the impulse responses of the filters on the output of seismometers. TFIRST is the time before which all impulse responses are zero. DELTAT is the time between adjacent time samples. For straight sum pattern studies one can use NFP = 1, TFIRST = 0, DELTAT = 0.

The next NS cards contain $Y(I)$, $X(I)$, $I = 1, \ldots, NS$ in the format (42X, 2F1Ø.3). The variables $Y(I)$, $X(I)$ are the north and east components of the position of the Ith point in the array.

In general, a sequence of NS groups of cards must follow. The Ith group contains $W(I, J)$, $J = 1, \ldots, NFP$ which is the impulse response, starting from TFIRST of the Ith filter, which is used on the Ith channel. The format is (5F12.7). For pattern studies all of these cards are omitted. Instruction 39 and the GO TO 39

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instruction set all filter weights to zero or unity as required. These commands must be removed to do other than straight sum pattern studies.

The next data card contains NTHETA, NFREQ, NK, DELTAK in format (3I5, F1Ø.5). These variables were described in the preceding section.

The time required to compute and plot 1000 points along a ray in wave number space for an array with 25 seismometers has been less than .5 minutes using an IBM 7094.

```
C      THREE DIMENSIONAL FOURIER TRANSFORM---LACOSS AUGUST 31,1965
C      MODIFIED 9/22/65
       DIMENSION SNTBL(2401),CSTBL(2401),X(600),Y(600),T(50),W(25,50)
C      NS--NUMBER OF SEISMOMETERS
C      NFP--NUMBER OF POINTS IN TIME FILTERS
C      TFIRST--TIME OF FIRST OUTPUT FROM FILTER WITH IMPULSE APPLIED
C              AT TIME ZERO
C      DELTAT--TIME BETWEEN FILTER SAMPLES
C      X(I),Y(I)--X AND Y COORD OF ITH SEISMOMETER (KM)
C      T(I)--TIME AT ITH SAMPLE IN TEMPORAL IMPULSE RESPONSES
C      W(I,J)--WEIGHT OF ITH SEISMOMETER AT JTH TIME (IMPULSE RESPONSE)
C      NTEST--CONTROLS PROGRAM TERMINATION (VALUE SET BY UTIL)
C      TRIVIAL CHANGES OF DIMENSION STATEMENTS WOULD ALLOW MORE OR LESS
C              SEISMOMETERS AND/OR FILTER POINTS IF MEMORY IS AVAILABLE
C      GAINR--REAL PART OF RESPONSE TRANSFORM
C      GAINI--IMAGINARY PART OF RESPONSE TRANSFORM
C      NRUNS--NUMBER OF DATA DECKS SUBMITTED
       WRITE (6,700)
700    FORMAT(55H1THREE DIMENSIONAL FOURIER TRANSFORM--LACOSS AUG31,1965)
C
C      FORM SIN AND COS TABLES
       DO 10 I=1,1200
       FLI=I
       THETA=(3.141592653/1200.)*(FLI-1.)
10     SNTBL(I)=SIN(THETA)
       DO 12 I=1201,2401
12     SNTBL(I)=-SNTBL(I-1200)
       DO 14 I=1,1801
14     CSTBL(I)=SNTBL(I+600)
       DO 16 I=1802,2401
16     CSTBL(I)=SNTBL(I-1800)
C      SIN COS TABLE IS COMPLETE
C
C      INITIALIZE DATA
       READ(5,599)NRUNS
599    FORMAT(I5)
       DO 500 KKKK=1,NRUNS
       READ (5,600) NS,NFP,TFIRST,DELTAT
600    FORMAT(2I5,2F10.5)
       WRITE (6,701) NS,NFP,TFIRST,DELTAT
701    FORMAT(4HONS=I3,10X,4HNFP=I3,10X,7HTFIRST=F9.5,10X,7HDELTAT=F9.5)
       READ (5,601) (Y(I),X(I),I=1,NS)
601    FORMAT(42X,2F10.3 )
       WRITE (6,702) (Y(I),X(I),I=1,NS)
702    FORMAT(36HOSEISMOMETER LOCATIONS N-S,E-W IN KM(2F20.3,10X,2F20.3))
       DO 30 I=1,NFP
       FLI=I
30     T(I)=TFIRST+DELTAT*(FLI-1.)
       DO 40 I=1,NS
       GO TO 39
       READ(5,602) (W(I,J),J=1,NFP)
602    FORMAT(5F12.7)
39     W(I,1)=1.
40     WRITE (6,703) I,(W(I,J),J=1,NFP)
703    FORMAT(20H0IMPULSE RESPONSE OFI3,17HTH SEISMOMETER I5/(10F10.5))
       NFLAG=1
C      END OF INITIALIZE


C      TRANSFORM LOOP
55     GO TO(50,50,500),NFLAG
50     CALL UTIL(XK,YK,FREQ,GAINR,GAINI,NFLAG,1)
       GAINR=0.
       GAINI=0.
       DO 200 I=1,NFP
       DO 200 J=1,NS
       ARG=-XK*X(J)-YK*Y(J)+FREQ*T(I)
       ARG=AMOD(ARG,1.)
       IF(ARG)60,65,65
60     ARG=ARG+1.
65     ARG =2400.*ARG
       K=IFIX(ARG)+1
       KK=K+1
       RATIO=AMOD(ARG,1.)
       COSVAL=CSTBL(K)+RATIO*(CSTBL(KK)-CSTBL(K))
       SINVAL=SNTBL(K)+RATIO*(SNTBL(KK)-SNTBL(K))
       GAINR=GAINR+W(J,I)*COSVAL
200    GAINI=GAINI+W(J,I)*SINVAL
       CALL UTIL(XK,YK,FREQ,GAINR,GAINI,NFLAG,2)
       GO TO 55
500    CONTINUE
       CALL PLTND
       STOP
       END
```

```fortran
      SUBROUTINE UTIL(XK,YK,F,GAINR,GAINI,NFLAG,INOUT)
C     SEPT 2,1965
C     ROUTINE SUPPLIES XK,YK,F TO FOURIER TRANSFORMER AND
C           OUTPUTS VALUES OF TRANSFORMS
      DIMENSION THETA(20),FREQ(20),STO1(1000),STO2(1000)
      GO TO (1,100),INOUT
    1 GO TO (5,50,50),NFLAG
C     INITIALIZE ON FIRST ENTRY
    5 WRITE (6,700)
  700 FORMAT(33HOUSED UTILITY PROGRAM SEPT 2,1965)
      READ (5,600) NTHETA,NFREQ,NK,DELTAK
  600 FORMAT(3I5,F10.5)
      WRITE (6,701) NTHETA,NFREQ,NK,DELTAK
  701 FORMAT(1UX,7HNTHETA=I5,9X,6HNFREQ=I5,9X,3HNK=I5,9X,7HDELTAK=F10.5)
C     NTHETA--NUMBER OF VALUES OF CLOCKWISE ANGLES FROM NORTH (THETA)
C           THAT ARE TO BE USED
C     NFREQ--NUMBER OF FREQUENCY (F OR FREQ) TO BE USED
C     NK--NUMBER OF RADIAL VALUES OF WAVE NUMBER TO BE USED
C     DELTAK--DIFFERENCE BETWEEN ADJACENT RADIAL VALUES OF WAVE NUMBER
      READ (5,601) (THETA(I),I=1,NTHETA)
      READ (5,601) (FREQ(I),I=1,NFREQ)
  601 FORMAT(8F10.5)
      WRITE (6,702) (THETA(I),I=1,NTHETA)
  702 FORMAT(16HOVALUES OF THETA/(10F10.5))
      WRITE (6,703) (FREQ(I),I=1,NFREQ)
  703 FORMAT(15HOVALUES OF FREQ/(10F10.5))
      ITHETA=1
      IFREQ=1
      IK=1
      NFLAG=1
      NFLAG=2
C     INITIALIZATION IS COMPLETE
C
C     GENERATE XK,YK,F CALLED BY MAIN PROGRAM
   50 IF(IK-1) 55,55,60
   55 TEMP=THETA(ITHETA)
      CTHETA=COS(TEMP)
      STHETA=SIN(TEMP)
   60 SIZEK=(FLOAT(IK)-1.)*DELTAK
      XK=STHETA*SIZEK
      YK=CTHETA*SIZEK
      F=FREQ(IFREQ)
      RETURN
C     XK,YK,F HAVE BEEN GENERATED
C
C     ENTRY POINT FOR INOUT=2
  100 STO1(IK)=GAINR
      STO2(IK)=GAINI
      IF(NK-IK) 110,110,150
  110 CALL  OUTPUT(STO1,STO2,NFLAG,NK,DELTAK)
      NFLAG=2
      IF(NTHETA-ITHETA) 115,115,140
  115 IF(NFREQ-IFREQ) 120,120,130
  120 NFLAG=3
      RETURN
  130 IK=1
      ITHETA=1
      IFREQ=IFREQ+1
      RETURN
  140 IK=1
      ITHETA=ITHETA+1
      RETURN
  150 IK=IK+1
      RETURN
      END
```

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```
      SUBROUTINE OUTPUT(STO1,STO2,MFLAG,NK,DELTAK)
C     SEPT 2,1965
C     MODIFIED 9/22/65
C     PLOTS TRANSFORMS OF 3DIM FUNCTIONS
      DIMENSION STO1(1000),STO2(1000)
      DIMENSION XCOOR(1000)
      GO TO (1,5),MFLAG
    1 READ (5,600) NCTRL,STO1U,STO1L
  600 FORMAT(I5,2F10.5)
      WRITE (6,700) NCTRL,STO1U,STO1L
  700 FORMAT(15HOOUTPUT CONTROL//7H NCTRL=,I5,9X,6HSTO1U=F10.5,9X,6HSTO1L=
     1F10.5)
      CALL STO1U9(20HGAIN AND PHASE PLOTS,4)
      CALL LAMKA,(9)
      DO 1020 J=1,NK
 1020 XCOOR(J)=FLOAT(J-1)*DELTAK
C     NCTRL--OUTPUT CONTROL
C          1--PLOT GAIN,PHASE
C          2--PLOT LOGGAIN,PHASE
C          3--1 AND PRINT SAME
C          4--2 AND PRINT SAME
C     STO1U--UPPER CLIP LEVEL FOR STO1
C     STO1L--LOWER CLIP LEVEL FOR STO1
C     MAX,MIN FOR STO2 ARE +PI AND -PI
    5 DO 10 I=1,NK


      TEMP=STO1(I)**2+STO2(I)**2
      GAIN=SQRT(TEMP)
      TEMP1=STO1(I)
      TEMP2=STO2(I)
      PHASE=ATAN2(TEMP2,TEMP1)
      STO1(I)=GAIN
      IF(I.EQ.1) DC=STO1(I)
      STO1(I)=STO1(I)/DC
   10 STO2(I)=PHASE
      WRITE(6,702) DC
  702 FORMAT(23H0ZERO WAVE NUMBER GAIN=F20.7)
      GO TO (40,20,30,20),NCTRL
   20 DO 25 I=1,NK
   25 TEMP=STO1(I)
      STO1(I)=20.*ALOG(TEMP)
      GO TO (40,40,30,30),NCTRL
   30 WRITE (6,701) (STO1(I),STO2(I),I=1,NK)
  701 FORMAT(///(2F10.5,9X,2F10.5,9X,2F10.5,9X,2F10.5))
   40 CONTINUE
      DO 50 I=1,NK
      IF(STO1(I).GE.STO1U) STO1(I)=STO1U
   50 IF(STO1(I).LE.STO1L) STO1(I)=STO1L
      STO1(I)=STO1L
      CALL KWKPLT(XCOOR,STO1 ,NK,18H     WAVE NUMBER     ,18H         GAIN
     1       )
      CALL KWKPLT(XCOOR,STO2 ,NK,18H     WAVE NUMBER     ,18H         PHASE
     1       )
      MFLAG=2
      RETURN
      END
```

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APPENDIX C

USEFUL PATTERN SYMMETRIES AND INVARIANCES

A comparison between members of the same family of geometries or between different families is made a bit less difficult when certain symmetries of patterns are noted and invariances with certain changes of geometry are taken into account.

For any geometry whatsoever, it is true that

$$\left| H(k_x, k_y) \right| = \left| H(-k_x, -k_y) \right| .$$

This follows from substitution into (1) and noting that a number and its complex conjugate have the same magnitude.

Suppose that a geometry is transformed into itself by a rotation of α radians. Then the pattern of that array is transformed into itself by a rotation of $-\alpha$ radians. This is best seen by using polar coordinates k, θ for wave number and r_q, ψ_q for the location of the q^{th} element of the array. Using this notation, the expression for the pattern becomes

$$H(k, \theta) = \frac{1}{N} \sum_{q=1}^{N} \exp\left[-i 2\pi k r_q \cos(\theta - \psi_q) \right] .$$

Reproduction of the array by rotation of α radians means that

$$H(k, \theta) = \frac{1}{N} \sum_{q=1}^{N} \exp\left\{ -i 2\pi k r_q \cos\left[\theta - (\psi_q + \alpha) \right] \right\}$$

But

$$\cos[\theta - (\psi_q + \alpha)] = \cos[(\theta - \alpha) - \psi_q] .$$

Using this fact, one obtains

$$H(k, \theta) = H(k, \theta - \alpha)$$

as · as asserted.

The facts derived above imply the following results. Let α be the smallest rotation which transforms an array into itself. If π is an integral multiple of α, then the pattern for that array will repeat itself every α radians. If π is not an integral multiple of α, then the pattern for the array will repeat itself every $\alpha/2$ radians. The first of these results is true directly from the relation $H(k, \theta) = H(k, \theta - \alpha)$ derived above. The second requires use of the fact that $|H(k_x, k_y)| = |H(-k_x, -k_y)|$ for any geometry at all.

Finally, consider a geometry in which there is a ring of elements. Suppose that elements are uniformly spaced on that ring and that one element is found due north of the array center. A rotation of that ring by π radians leaves the pattern unchanged in the east and west directions in wave number space.

This fact is seen as follows. The function $H(k, \theta)$ is a one-dimensional transform of the array geometry projected onto a line in the θ direction. The rotation of a single ring as described above leaves the projection of the array onto a line in the east-west direction unchanged. Thus, $H(k, \theta)$ is not modified if θ is the angle which identifies

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either east or west. As a direct result of this invariance, one can conclude that patterns for geometries such as are shown in Fig. 6 must be the same in the east or west directions in wave number space. In the particular case, the same is true in directions an integral number of $\pi/5$ radians from east. This invariance with rotation can be useful when trying to compare geometries with an odd number of elements per ring with and without rotation as shown in Fig. 6. This invariance is not relevant when considering geometries having an even number of elements per ring such as those shown in Figure 7.

APPENDIX D

PROGRAM-SENSITIVITY SUBROUTINE

Plots of sensitivity of patterns with radii of rings have been obtained using the subroutine listed in this appendix. The routine is called by 3DFT which is listed in Appendix B. The pattern along a ray in wave number and the sensitivity along that ray for each ring are computed. This information is plotted out. This can be done for several directions in wave number space.

Use of the Sensitivity Programs

3DFT must be submitted as the main program to be used with the sensitivity routine. The output from the program as written requires the Stromberg Carlson 4020 plotter. If this is not available, the basic program remains valid but output instructions must be changed.

The first data card contains NRUNS in format (I5). This is the same variable described in Appendix B and indicates the number of sets of data to be processed.

Each set of data is constituted as follows. The first card contains NS, NFP, TFIRST, DELTAT in format (2I5, 2F10.5) with NFP = 1, TFIRST = 0, DELTAT = 0, and NS the number of seismometers or subarrays. The next NS cards contain north and east coordinates of subarrays or seismometers in a format (42X, 2F10.3). The next data card contains NTHETA, NK, NR, NPR, DELTAK in format (4I5, F10.5). NTHETA, NK, and DELTAK have the same meaning as in Appendix B. It is assumed that the array has at least NR rings of NPR evenly spaced seismometers or subarrays. The

sensitivity is to be computed for the radii of these NR rings. The next group of data is

RADIUS(I), I = 1,...,NR in format (8F1$\emptyset$.5). These are radii of rings of interest.

Finally, ANGLE(I), I = 1,...,NR in format (8F1$\emptyset$.5) are read. ANGLE(I) is the

orientation in radians of one element on the I^{th} ring, which has radius RADIUS(I).

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```
      SUBROUTINE UTIL(XK,YK,F,GAINR,GAINI,NFLAG,INOUT)
C     OCT 30,1965
C     ROUTINE SUPPLIES XK,YK,F TO FOURIER TRANSFORMER, COMPUTES
C     SENSITIVITY OF TRANSFORM MAGNITUDE WITH RADII OF RINGS OF
C     AN ARRAY, PLOTS SENSITIVITY AND TRANSFORM MAGNITUDE ALONG
C     A RAY IN WAVE-NUMBER SPACE
      DIMENSION THETA(20),STO1(1000),STO2(1000),SENS(1000),RADIUS(10)
      DIMENSION ANGLE(10),PSI(25),XCOOR(1000)
      GO TO(1,100),INOUT
    1 GO TO(5,50,50),NFLAG
C     INITIALIZE ON FIRST ENTRY
    5 WRITE(6,700)
  700 FORMAT(33HUSED UTILITY PROGRAM OCT 30,1965)
      READ(5,600)NTHETA,NK,NR,NPR,DELTAK
  600 FORMAT(4I5,F10.5)
      WRITE(6,701)NTHETA,NK,NR,NPR,DELTAK
  701 FORMAT(9TH VALUES OF NTHETA,NK,NR,NPR,DELTA ARE4I9,F10.5)
C     NTHETA--NUMBER OF RAYS IN WAVE-NUMBER SPACE
C     NK--NUMBER OF RADIAL VALUES ON EACH WAVE-NUMBER RAY
C     DELTAK--DIFFERENCE BETWEEN ADJACENT RADIAL VALUES OF WAVE-NUMBER
C     NR--NUMBER OF RINGS FOR WHICH SENSITIVITY COMPUTED
C     NPR--SEISMOMETERS PER RING OF THE ARRAY
      READ(5,601) (THETA(I),I=1,NTHETA)
      READ(5,601) (RADIUS(I),I=1,NR)
      READ(5,601) (ANGLE(I),I=1,NR)
  601 FORMAT(8F10.5)
C     THETA--ANGLE OF WAVE NUMBER RAYS(FROM NORTH)
C     RADIUS--RADII OF ARRAY RINGS OF INTEREST
C     ANGLE--ANGLE FROM NORTH TO AN ELEMENT OF ARRAY ON EACH RING
      WRITE(6,702) (THETA(I),I=1,NTHETA)
  702 FORMAT(16H0VALUES OF THETA/(10F10.5))
      WRITE(6,703) (RADIUS(I),I=1,NR)
  703 FORMAT(17H0VALUES OF RADIUS/(10F10.5))
      WRITE(6,704) (ANGLE(I),I=1,NR)
  704 FORMAT(16H0VALUES OF ANGLE/(10F10.5))
      ITHETA=1
      IK=1
      MFLAG=1
      NFLAG=2
      DPSI=2.03.1415926/FLOAT(NPR)
C     INITIALIZATION COMPLETE
C     GENERATE XK,YK,F CALLED BY MAIN PROGRAM
   50 IF(IK-1)55,55,60
   55 TEMP=THETA(ITHETA)
      CTHETA=COS(TEMP)
      STHETA=SIN(TEMP)
   60 SIZEK=(FLOAT(IK)-1.)*DELTAK
      XK=STHETA*SIZEK
      YK=CTHETA*SIZEK
      F=0.
      RETURN
C     XK,YK,F HAVE BEEN SUPPLIED
C
C     ENTRY POINT FOR INOUT=2
  100 STO1(IK)=GAINR
```

```
      STO2(IK)=GAINI
      IF(IMK-IK) 110,110,150
  110 GO TO(200,300),MFLAG
  200 CALL STOIDV(9HPATTERN SENSITIVITY WITH RING POSITIONS,5)
      CALL CAMRAV(9)
      DO 210 J=1,NK
  210 XCOOR(J)=FLOAT(J-1)*DELTAK
  300 DO 310 I=1,NK
      STOR1=STO1(I)
      STOR2=STO2(I)
      GAIN=SQRT(STOR1**2+STOR2**2)
      STO2(I)=ATAN2(STOR2,STOR1)
      IF(I.EQ.1) DC=GAIN
  310 STO1(I)=GAIN/DC
C     NOW STO1,STO2 ARE NORMALIZED GAIN,PHASE RESPECTIVELY
      STO1(1)=0.
      CALL RWKPLT(XCOOR,STO1,NK,18H    WAVE NUMBER    ,18H        GAIN
     1  )
      DO 390 IRING=1,NR
      DO 320 I=1,NK
  320 SENS(I)=0.
      DO 325 I=1,NPR
  325 PSI(I)=ANGLE(IRING)+(FLOAT(I)-1.)*DPSI
      DO 380 K=1,NK
      DO 360 IPR=1,NPR
      DIF=THETA(ITHETA)-PSI(IPR)
      COSDIF=COS(DIF)
      PHI=STO2(K)
      SIZEK=XCOOR(K)
      RAD=RADIUS(IRING)
  360 SENS(K)=SENS(K)-2.03.1415926*SIZEK*COSDIF*SIN(PHI+2.03.1415926*RAD
     1*SIZEK*COSDIF)
  380 CONTINUE
  390 CALL RWKPLT(XCOOR,SENS,NK,18H    WAVE NUMBER    ,18H    SENSITIVITY
     1  )
      MFLAG=2
      IF(NTHETA-ITHETA)115,115,140
  140 IK=1
      ITHETA=ITHETA+1
      RETURN
  115 NFLAG=3
      RETURN
  150 IK=IK+1
      RETURN
      END
```

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3-64-4334

RADIUS 100 km

O EXPERIMENTAL LASA
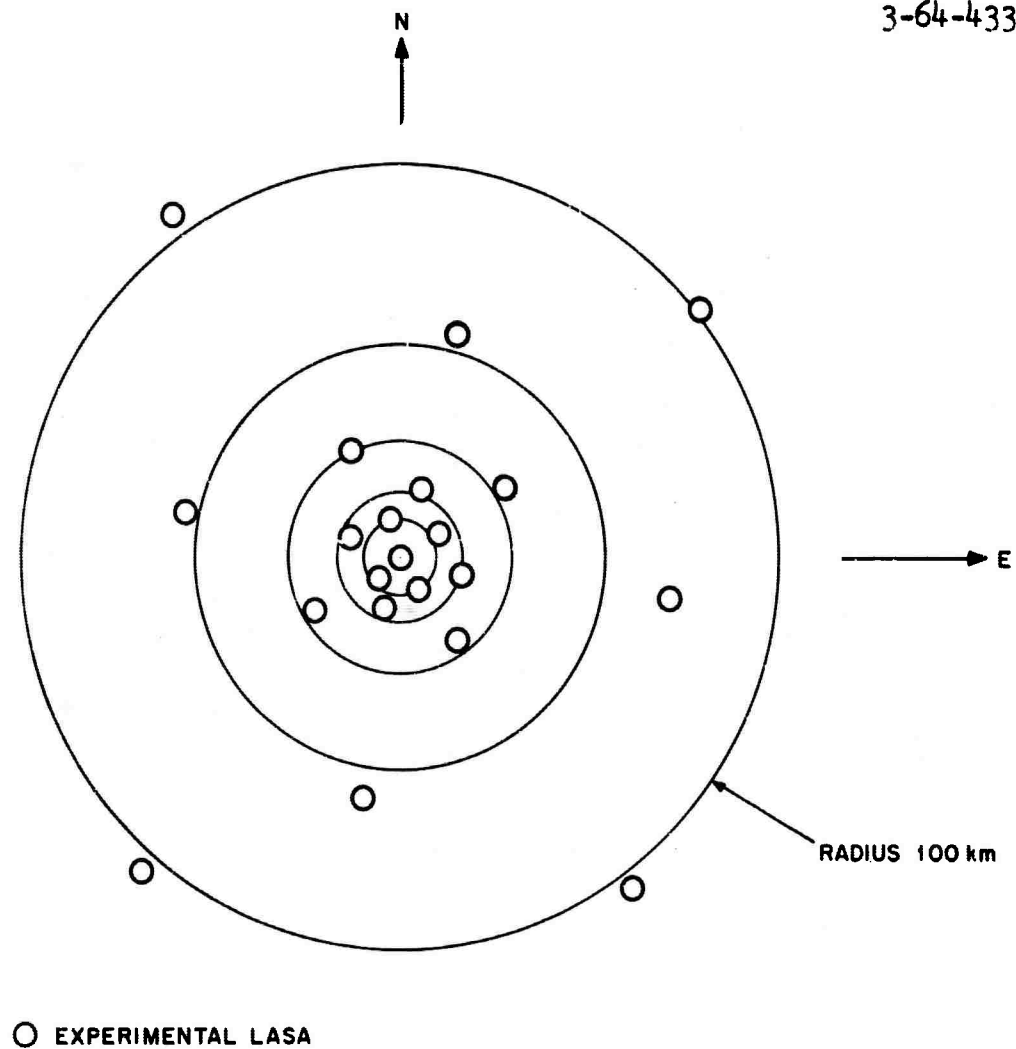
Figure 1   Configuration of experimental LASA.

50
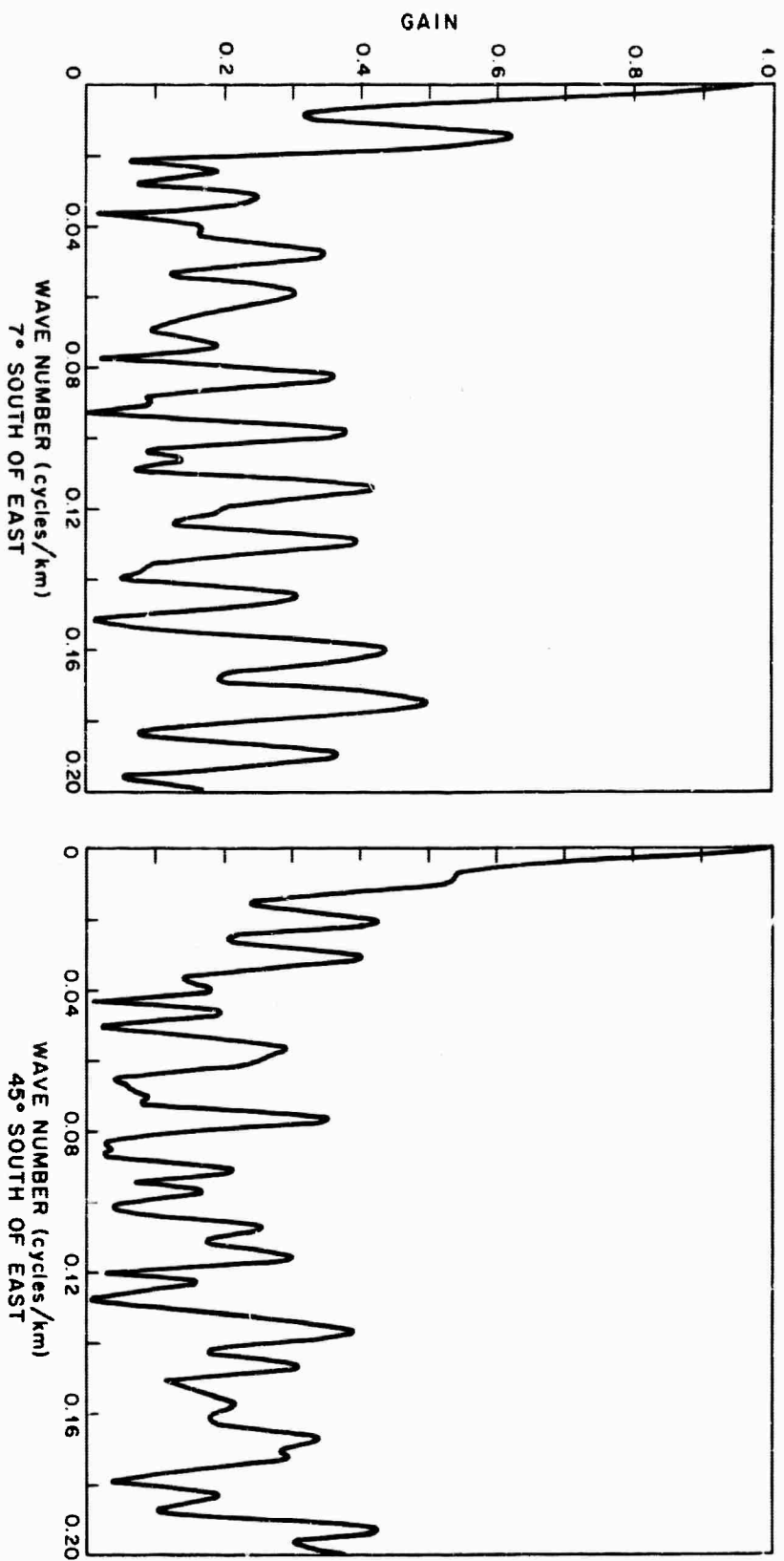
Figure 2   Array patterns (cross sections) of experimental LASA.

3-64-4335
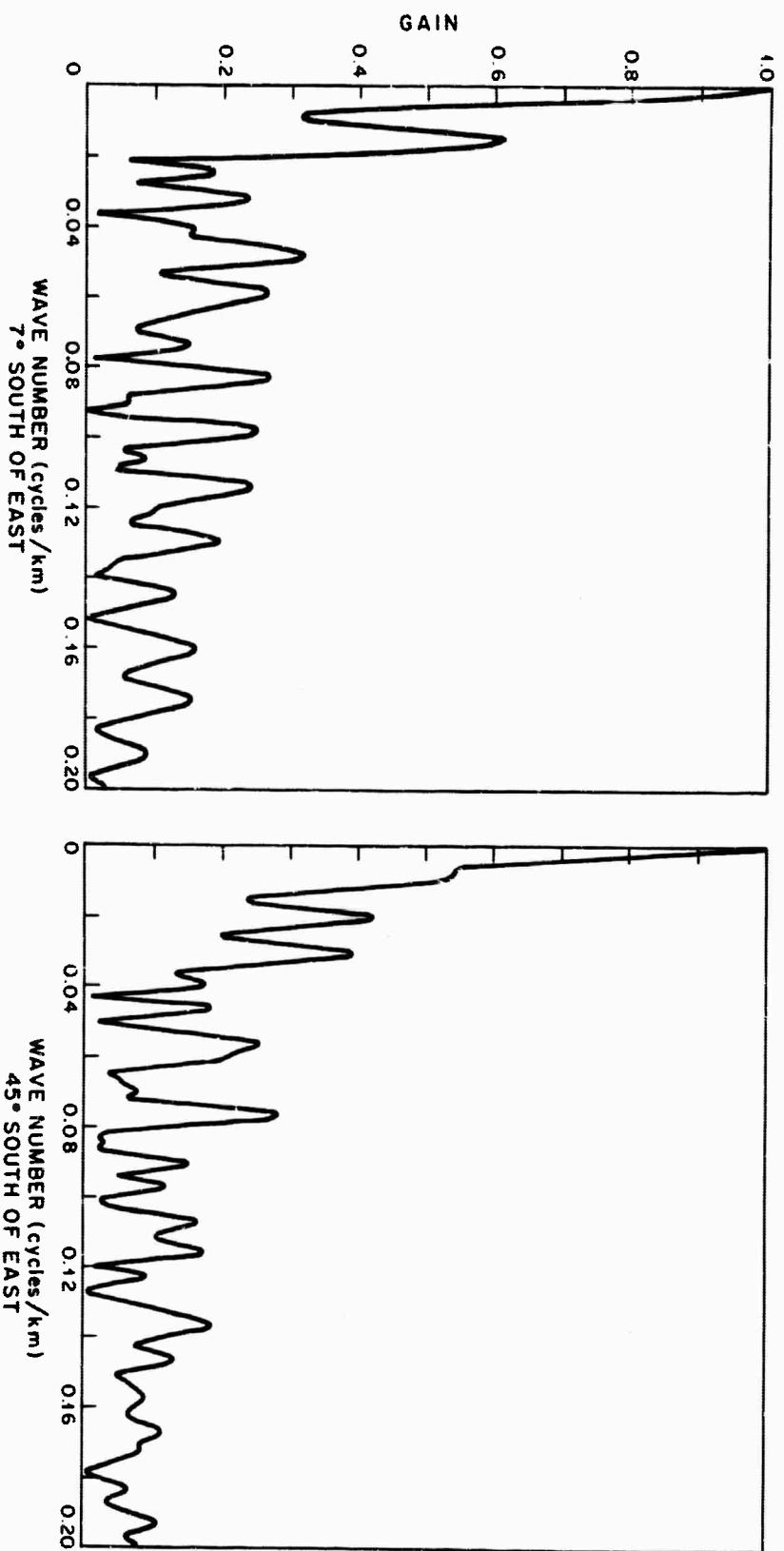
Figure 3   Full array patterns (cross sections) of experimental LASA.
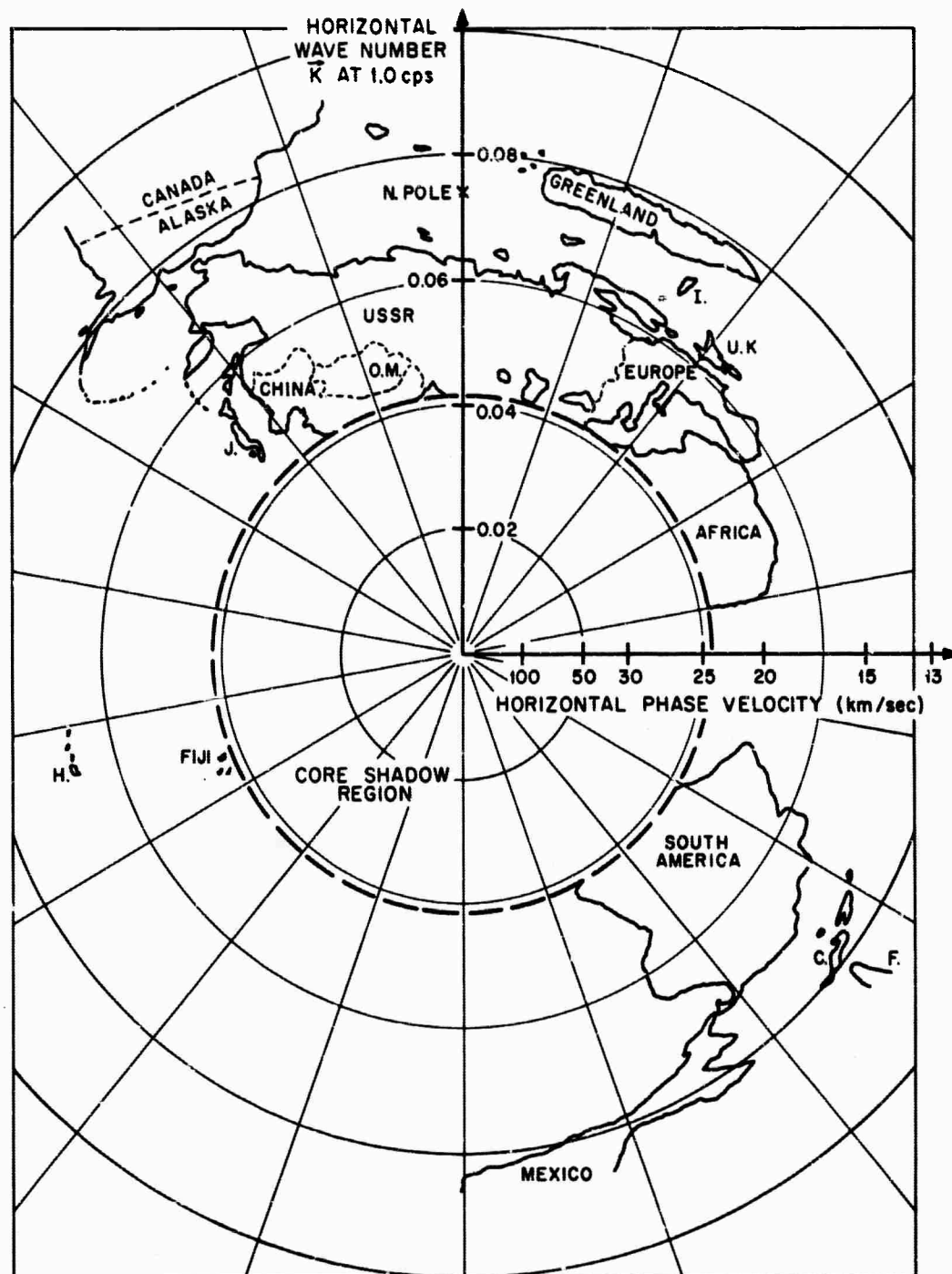
Figure 4   Earth as seen from center of experimental LASA.

3-64-4338
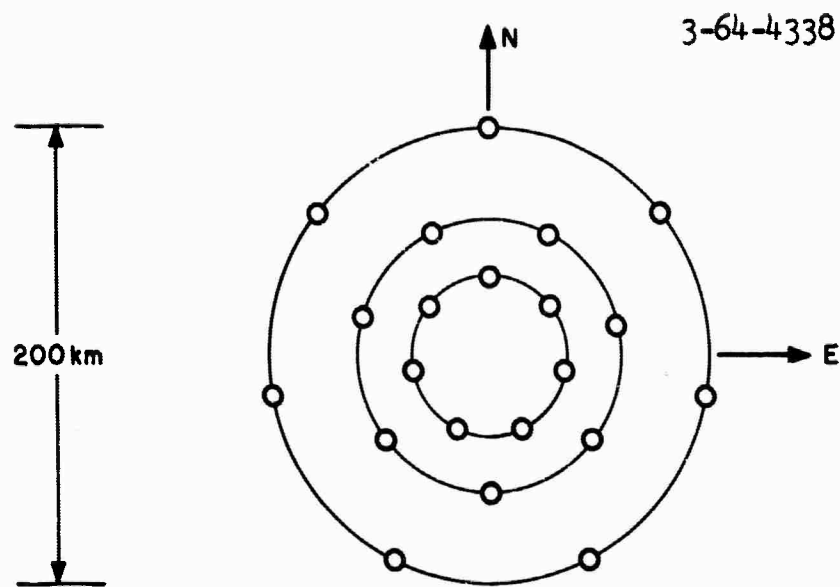
ADJACENT RINGS ROTATED RELATIVE TO EACH OTHER
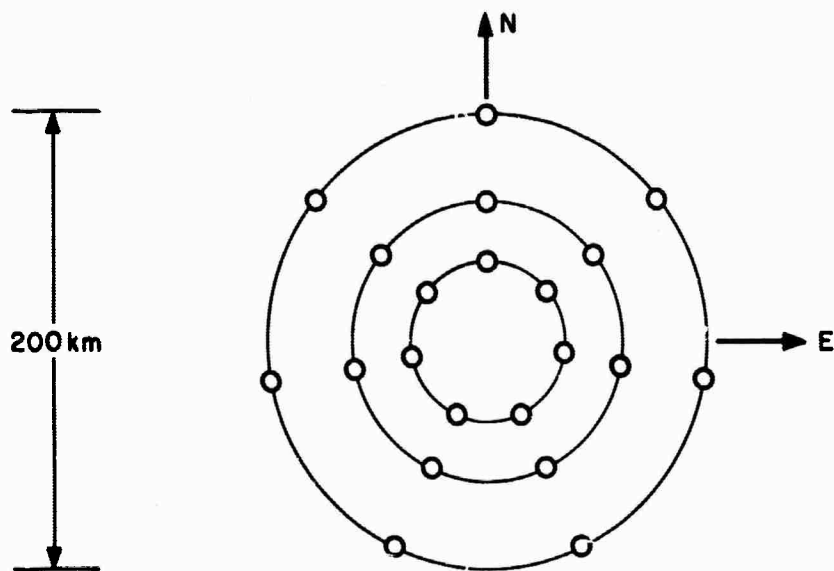
NO ROTATION BETWEEN ADJACENT RINGS

Figure 5   Array geometries having seven subarrays per ring.
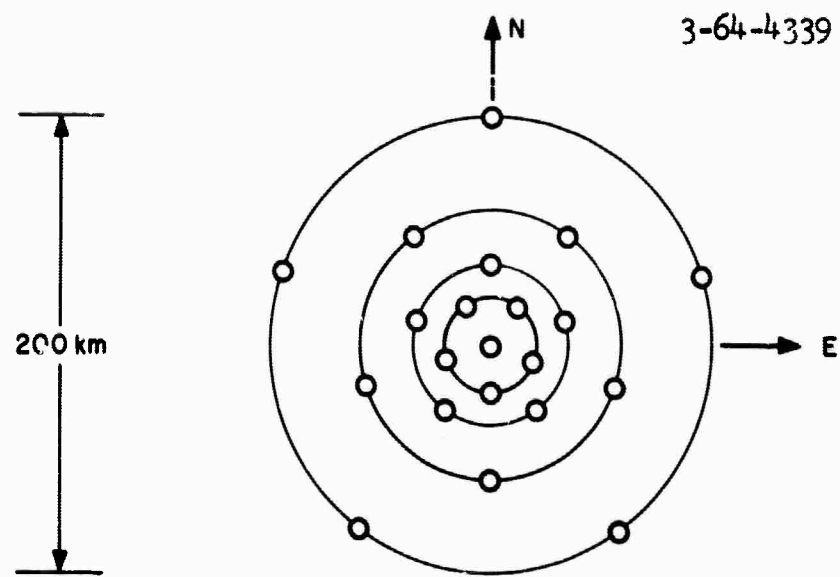
ADJACENT RINGS ROTATED RELATIVE TO EACH OTHER
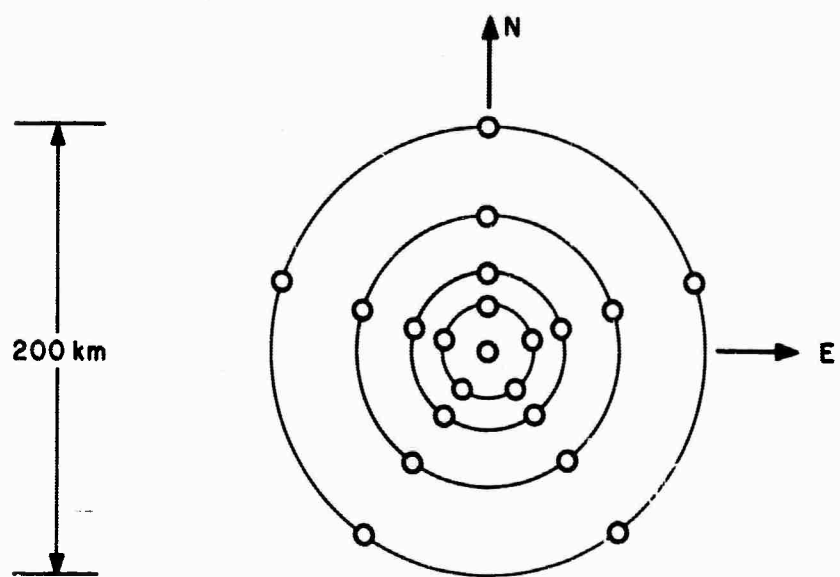
NO ROTATION BETWEEN ADJACENT RINGS

Figure 6   Array geometries having five subarrays per ring.
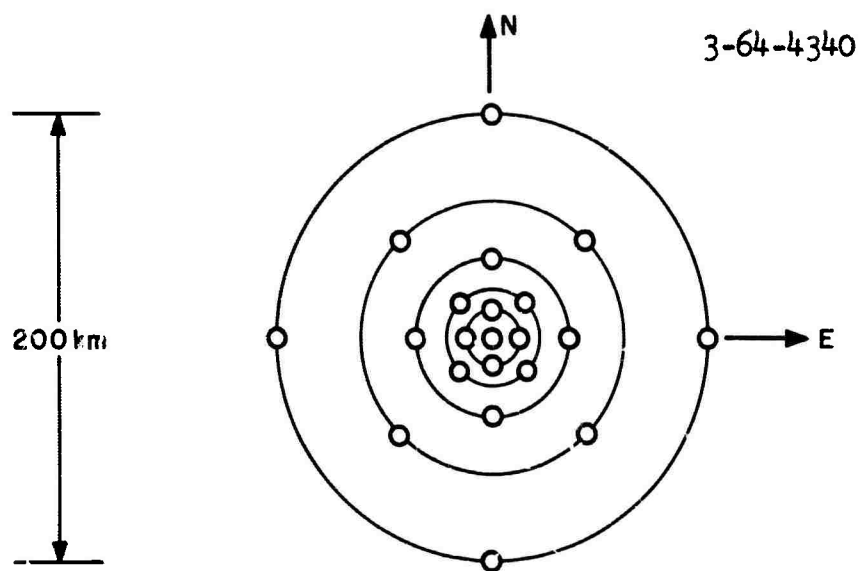
3-64-4340

ADJACENT RINGS ROTATED RELATIVE TO EACH OTHER
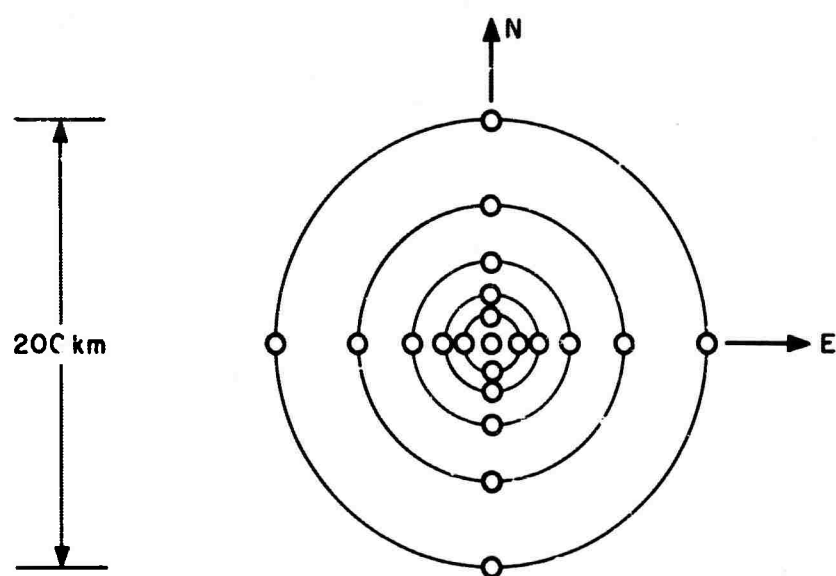
NO ROTATION BETWEEN ADJACENT RINGS

Figure 7   Array geometries having four subarrays per ring.

3-64-4341

ADJACENT RINGS ROTATED RELATIVE TO EACH OTHER

NO ROTATION BETWEEN ADJACENT RINGS
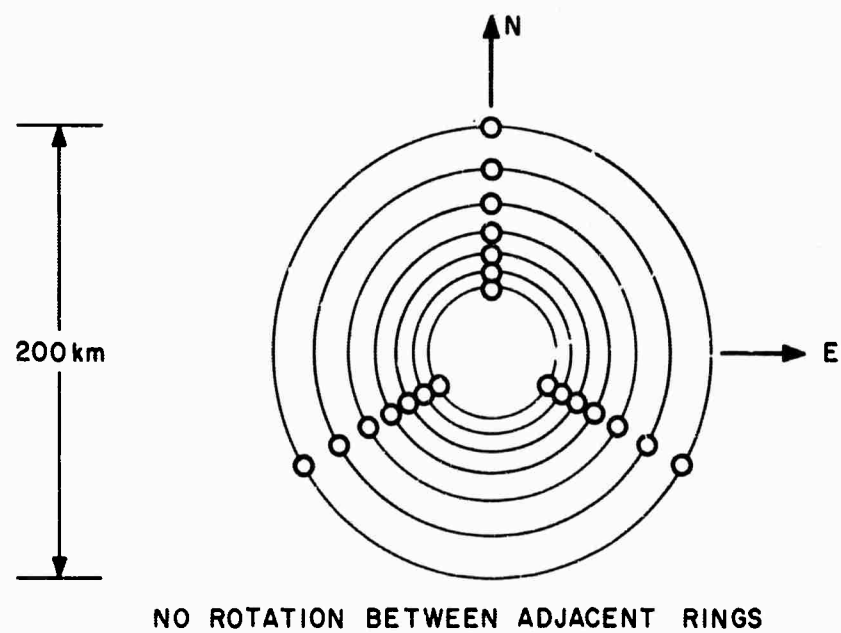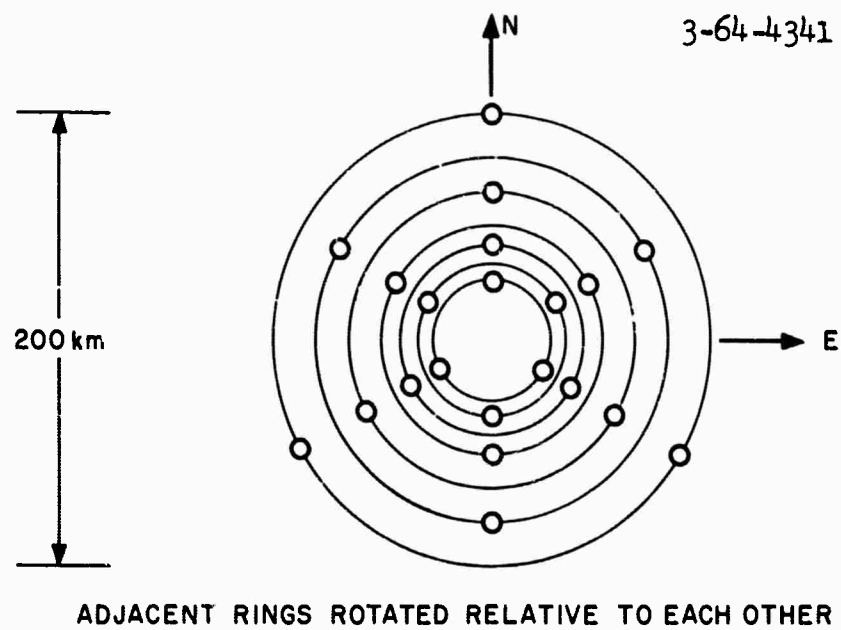
Figure 8   Array geometries having three subarrays per ring.

3-64-4342

N

RADIUS 100 km

RADIUS 70 km

E

RADIUS 49 km
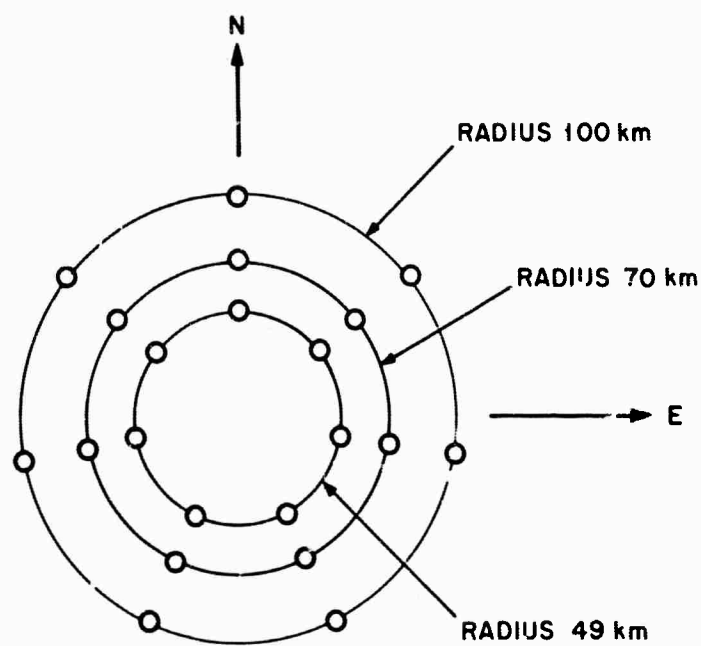
(A)   Array geometry.

Figure 9   Geometry and patterns for an array having seven subarrays per
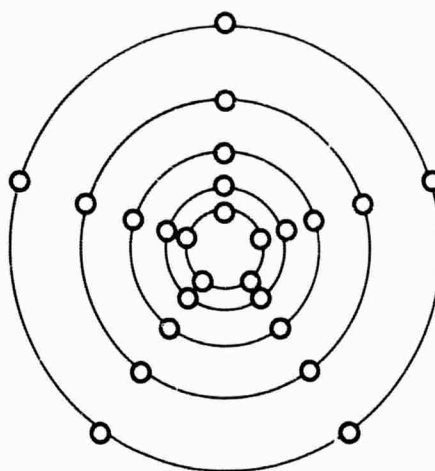concentric ring and geometric spacing of rings.
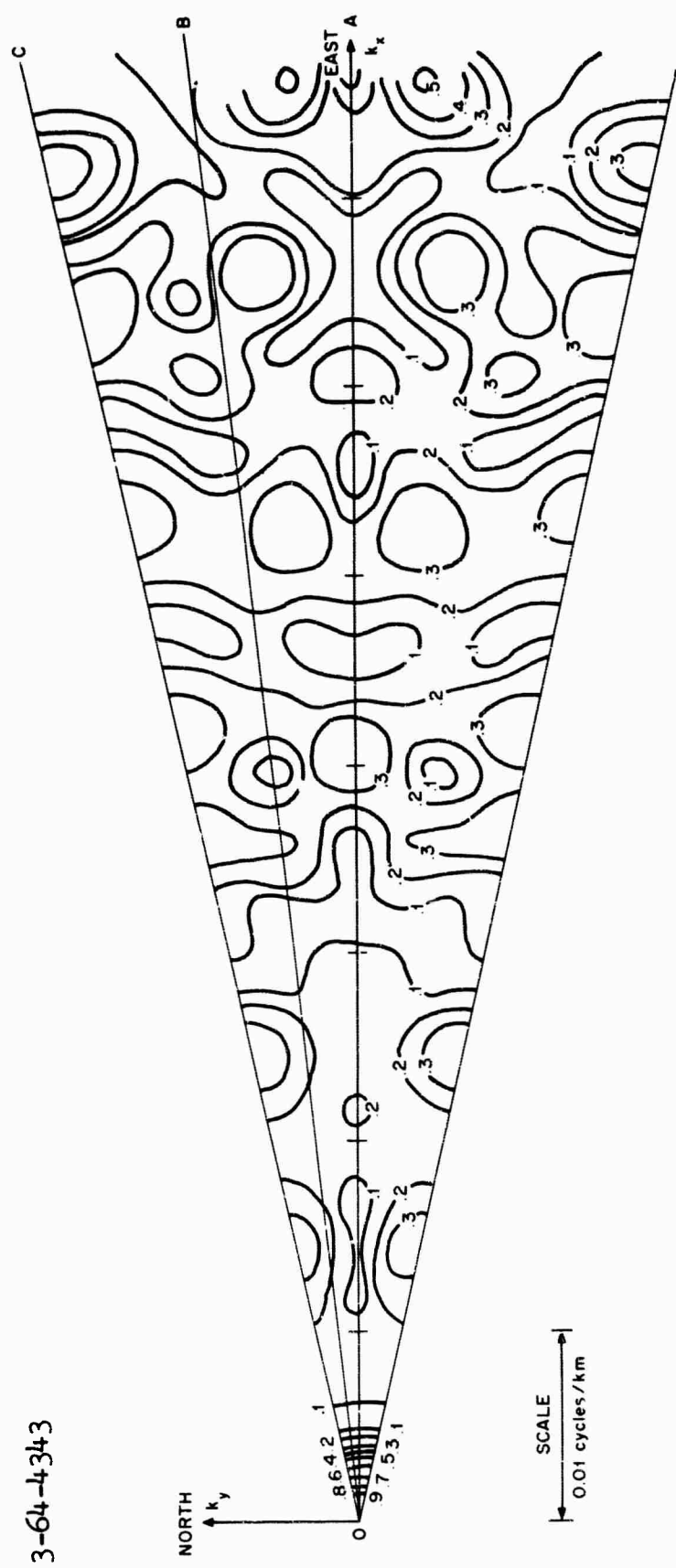
(B)   Pattern (contour map).

Figure 9 Continued.

3-64-4343

59
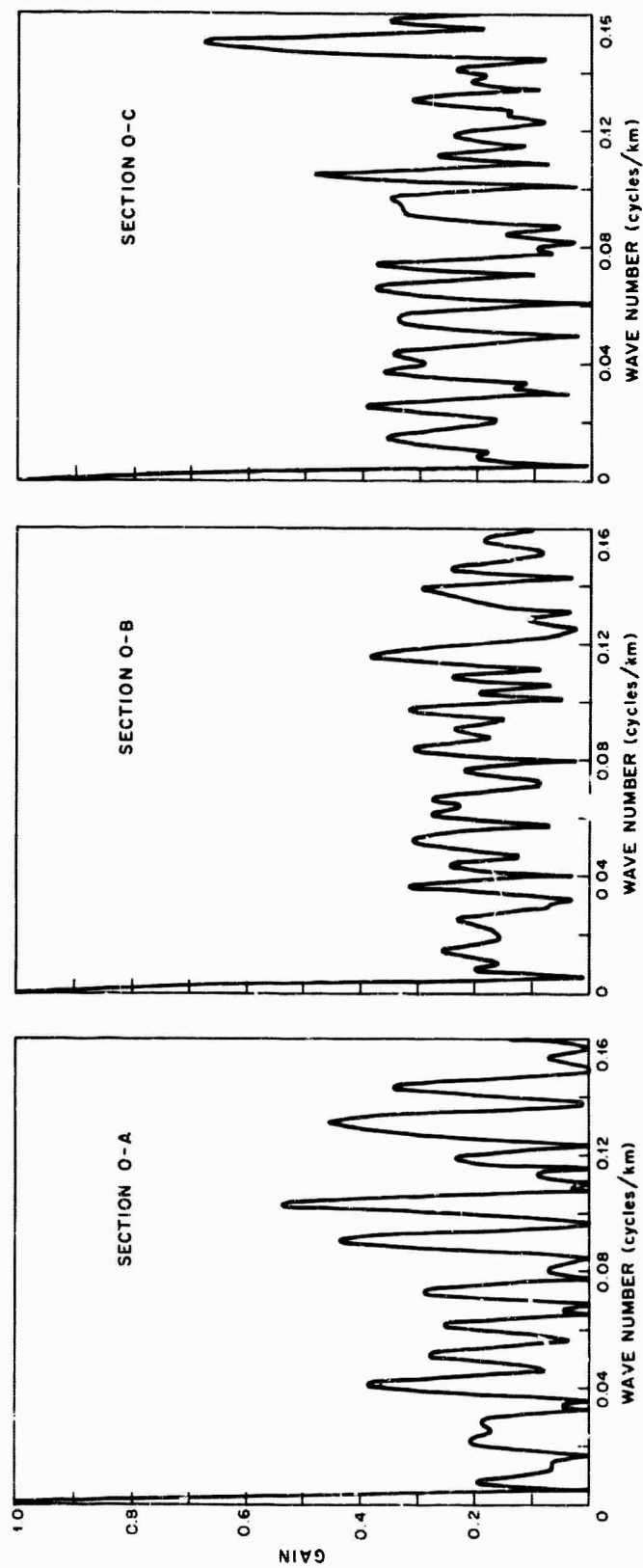
(C)   Pattern (cross sections).

Figure 9 Continued.

3-64-4345

N

RADIUS 100 km

RADIUS 40 km

E

RADIUS 16 km
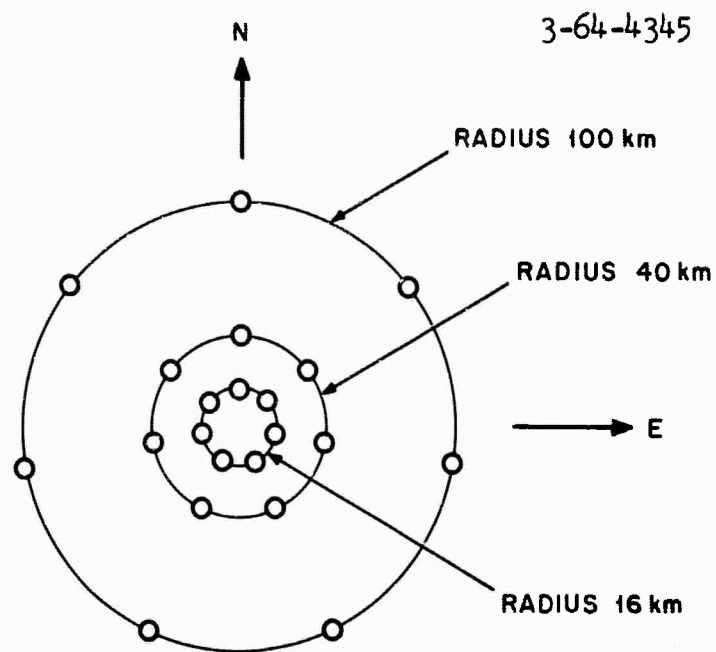
(A)   Array geometry.

Figure 10   Geometry and patterns for an array having seven subarrays per
concentric ring and geometric spacing of rings.

(B)   Pattern (contour map).
       Figure 10 Continued.

62
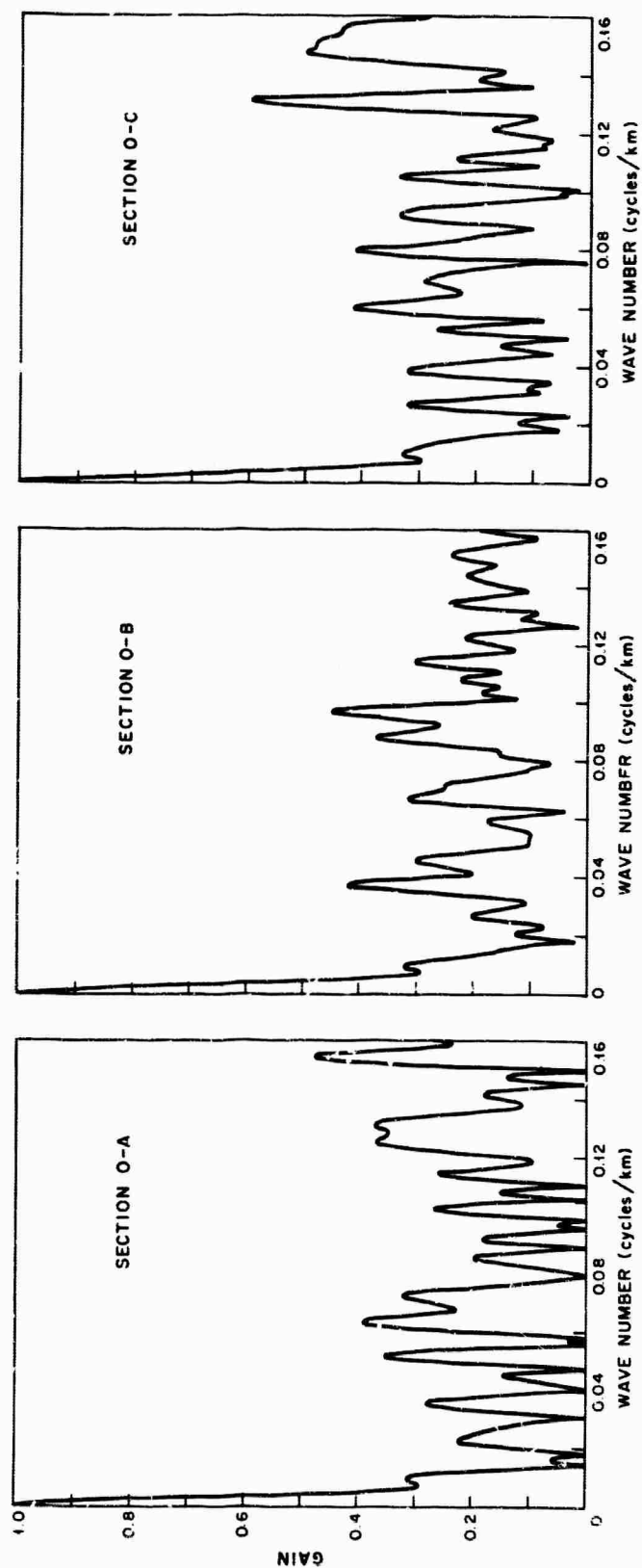
(C)   Pattern (cross sections).
Figure 10 Continued.

63

3-64-4348
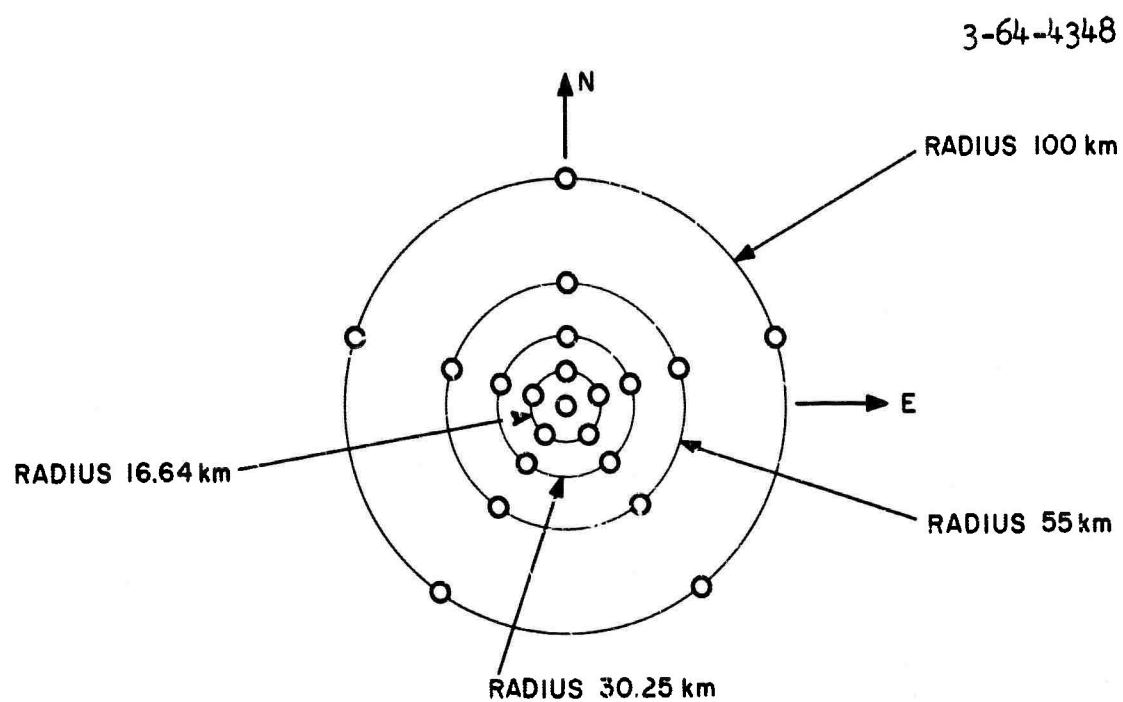
(A)   Array geometry.

Figure 11   Geometry and patterns for an array having five subarrays per
concentric ring and geometric spacings of rings.
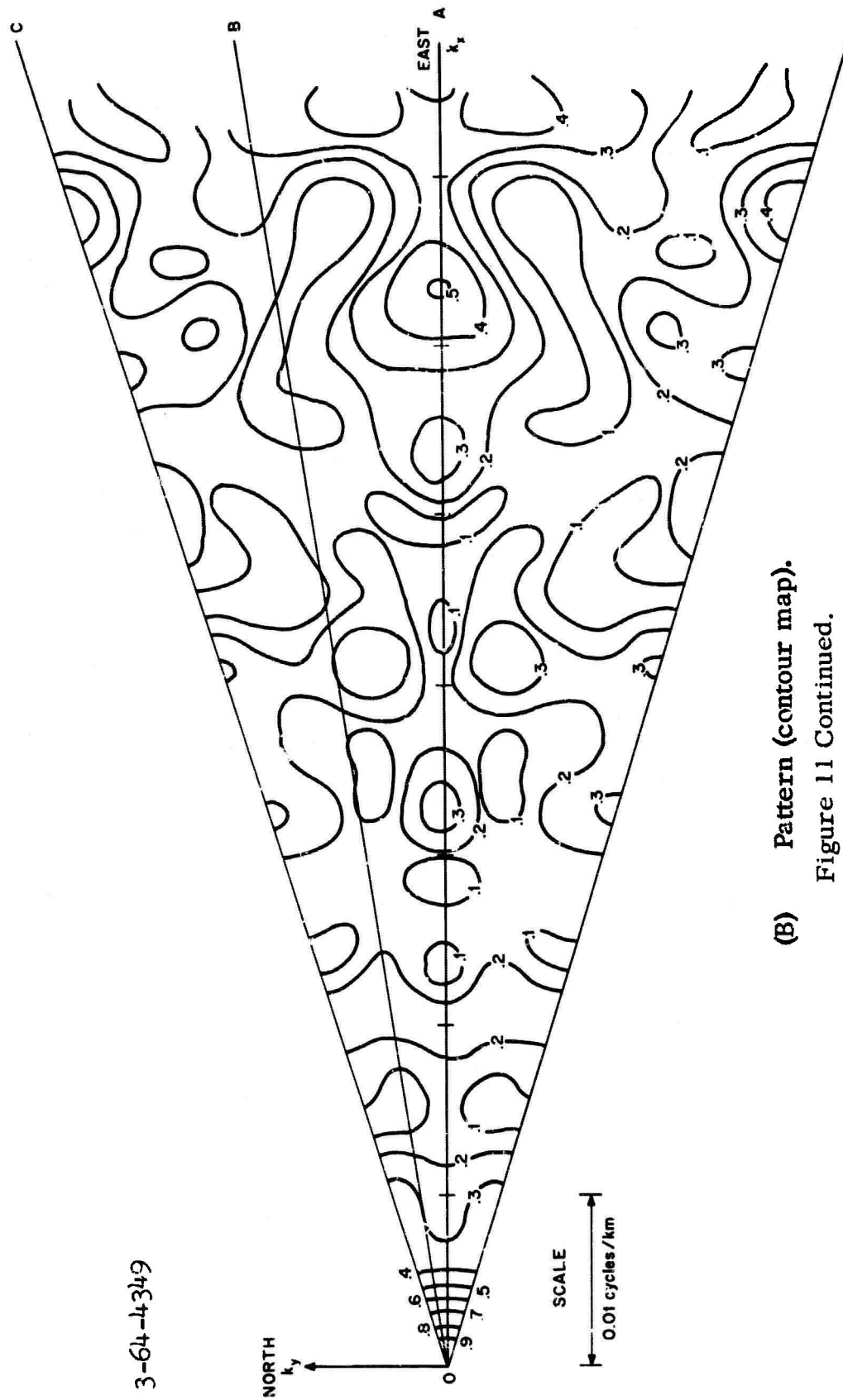
(B)   Pattern (contour map).
Figure 11 Continued.

65
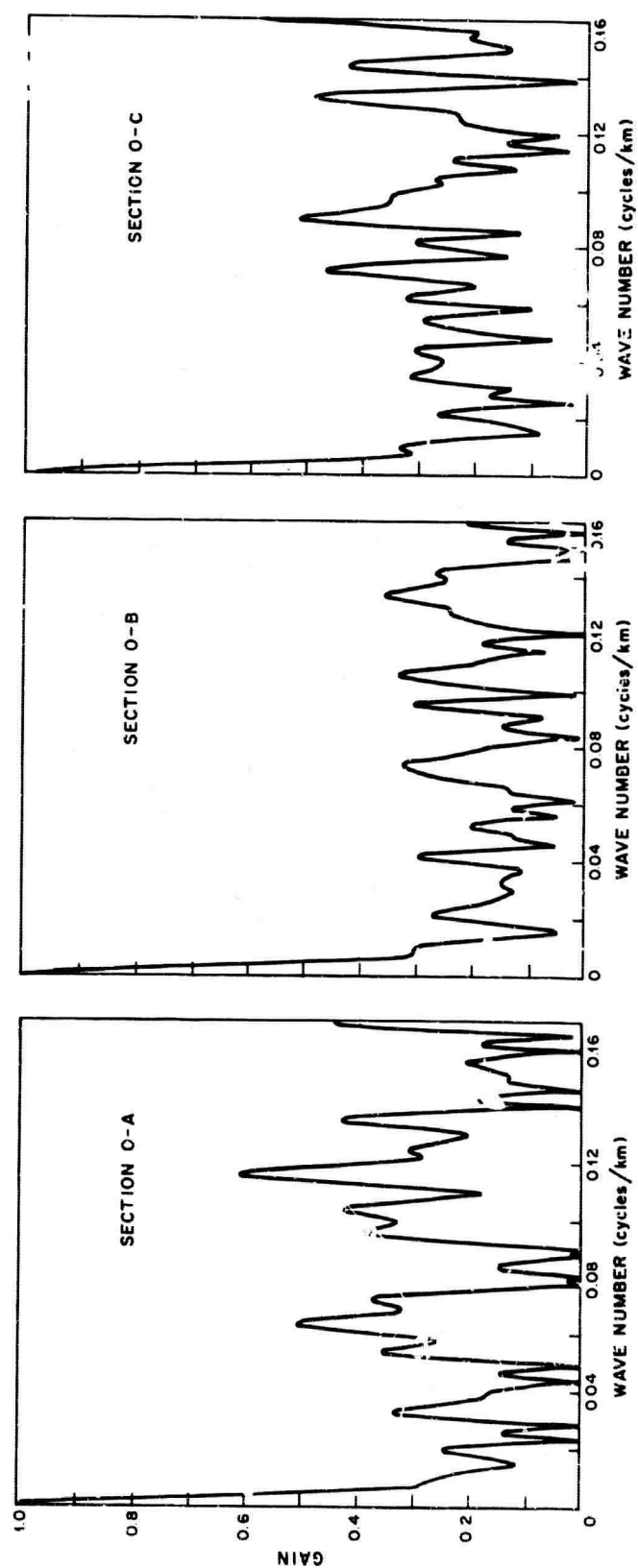
3-64-4350

(C) Pattern (cross sections).
Figure 11 Continued.
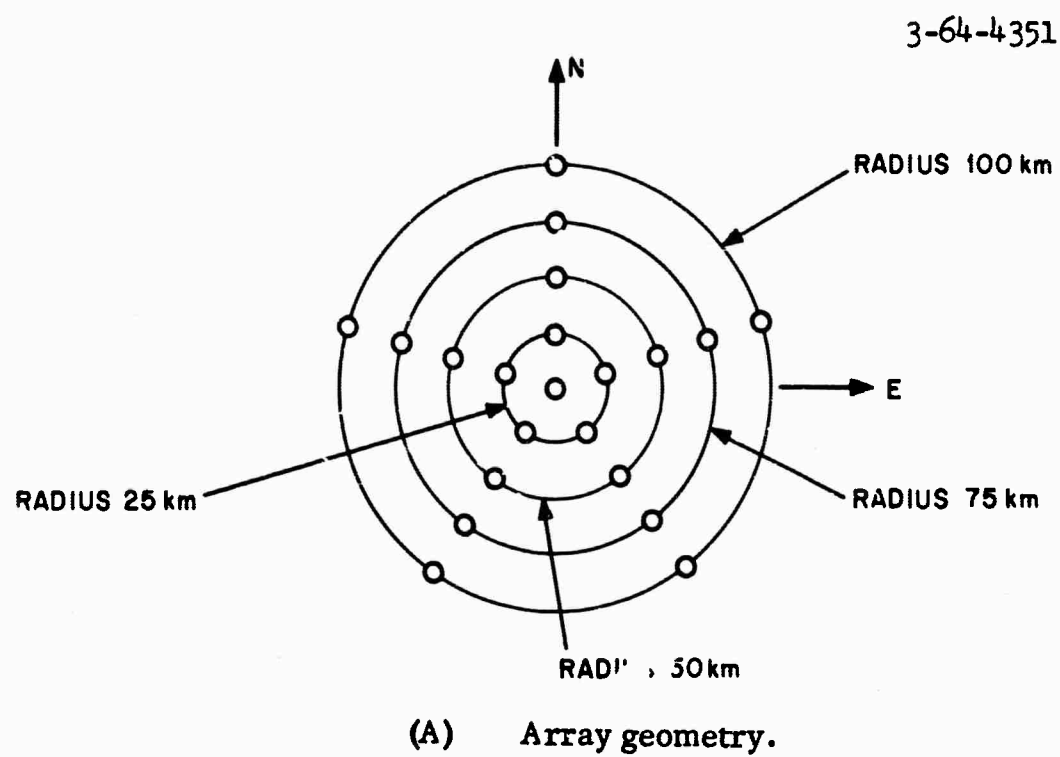
(A)   Array geometry.

Figure 12   Geometry and patterns for an array having five subarrays per concentric ring and arithmetic spacing of rings.
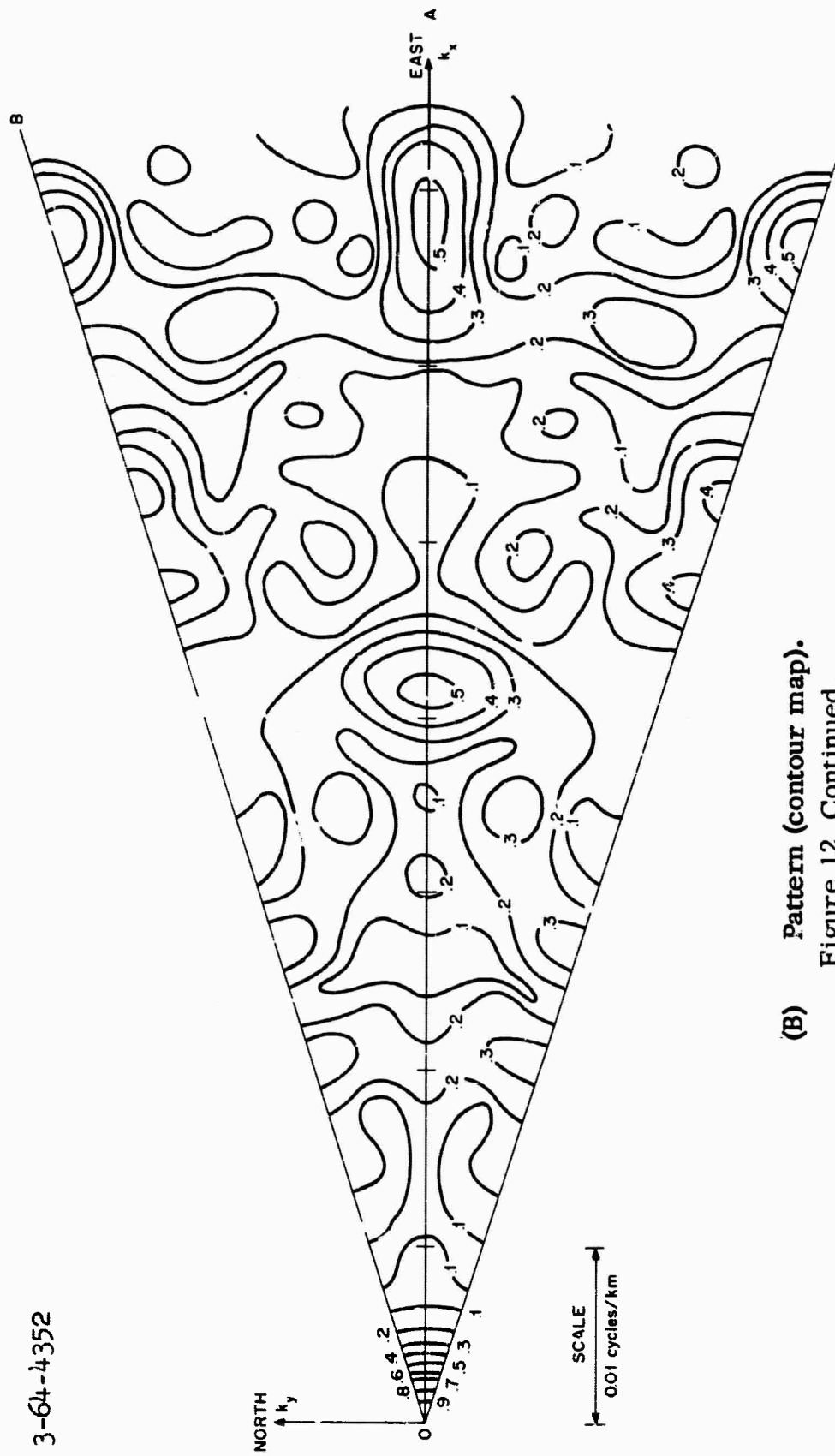
(B)   Pattern (contour map).
        Figure 12 Continued.

3-64-4352

SCALE
0.01 cycles/km
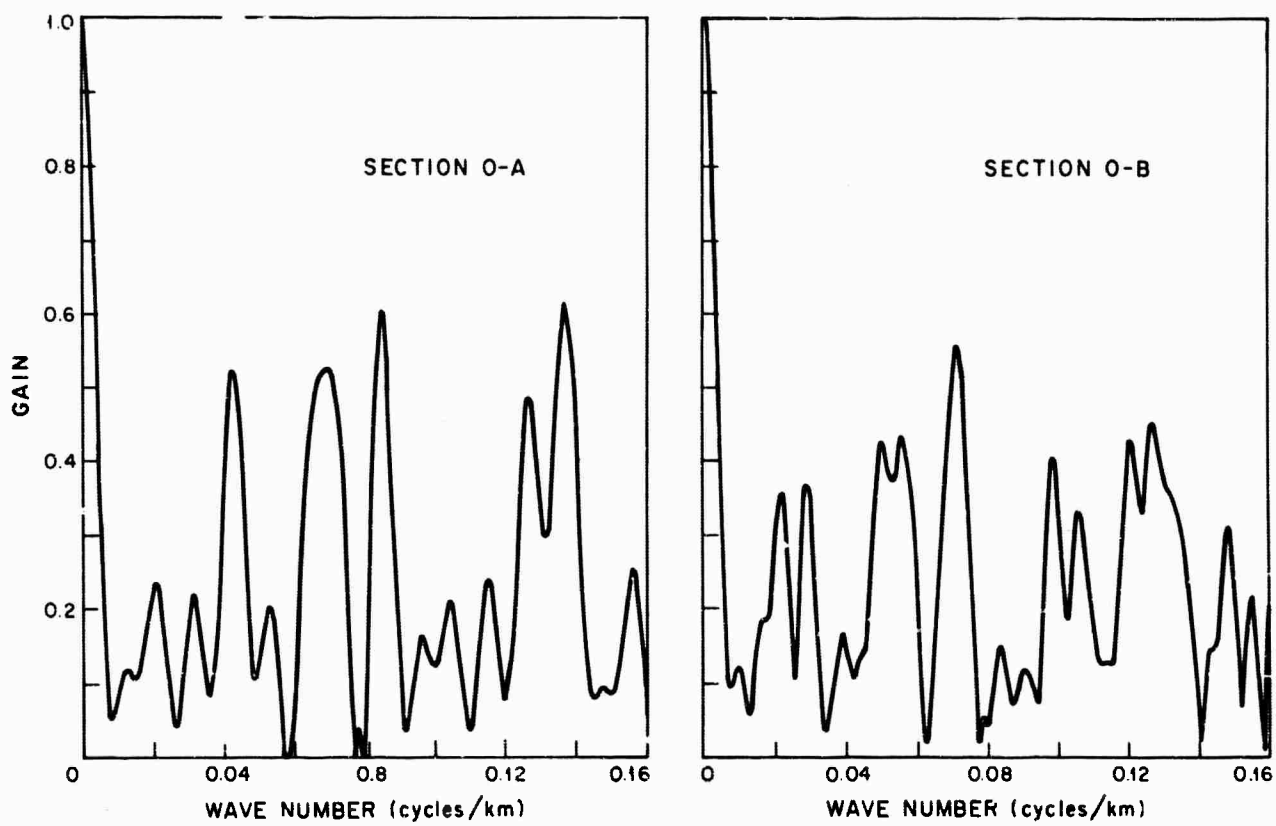
(C)   Pattern (cross sections).
Figure 12 Continued.

RADIUS 100 km

RADIUS 60 km

N

E

RADIUS 36 km

RADIUS 21.6 km
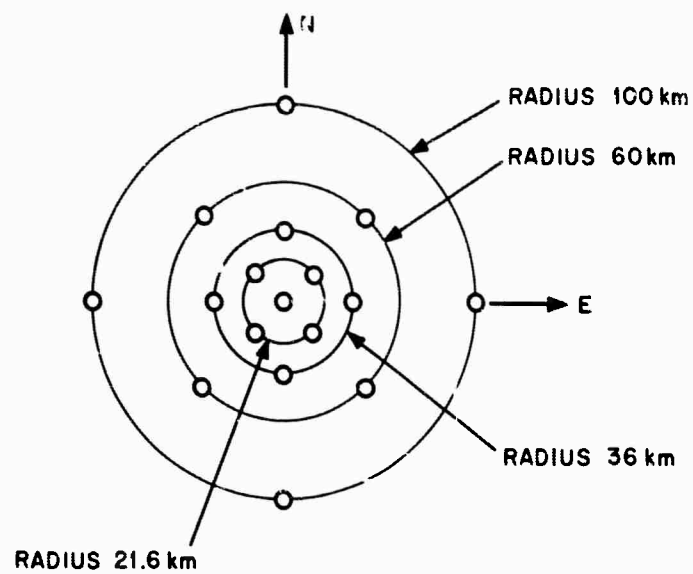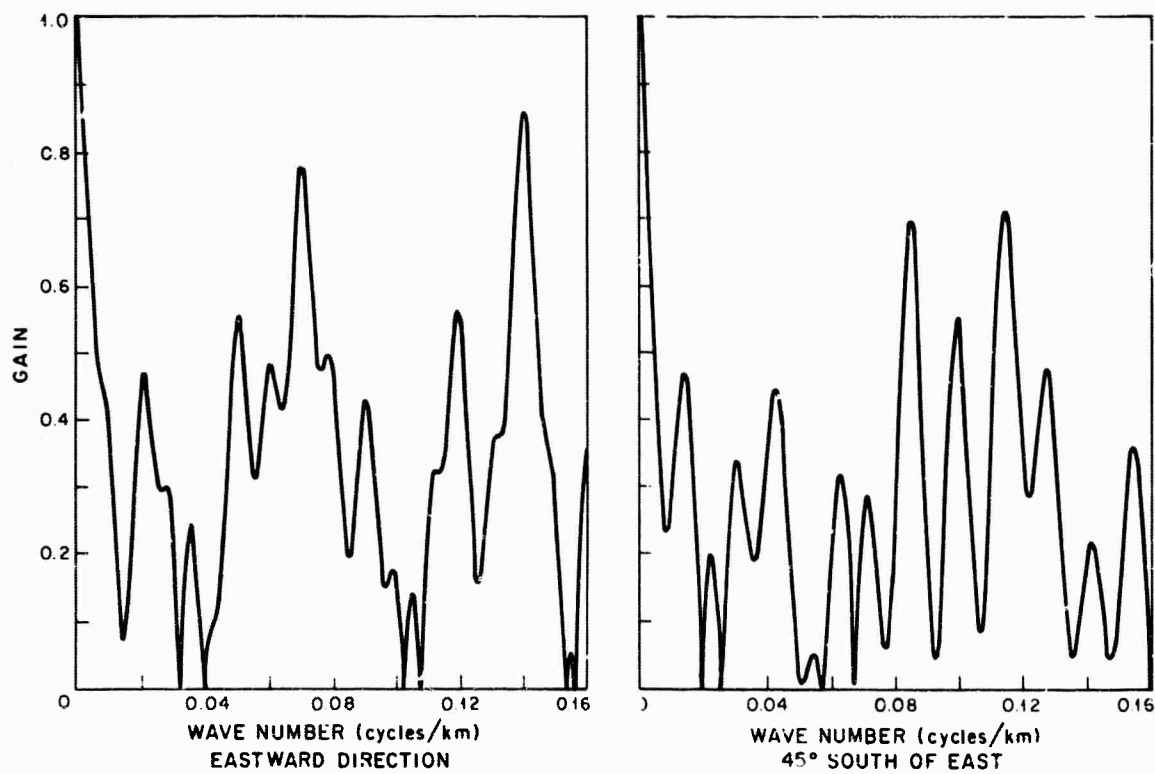
(A)   Array geometry.

(B)   Pattern (cross sections).

Figure 13   Geometry and patterns for an array having four subarrays per concentric ring and geometric spacing of rings.

N

RADIUS 11.77 km

RADIUS 100 km

RADIUS 70 km

E

RADIUS 16.81 km

RADIUS 49 km

RADIUS 24.01 km

RADIUS 34.3 km

(A)   Array geometry.

GAIN

WAVE NUMBER (cycles/km)
EASTWARD DIRECTION

WAVE NUMBER (cycles/km)
30° SOUTH OF EAST
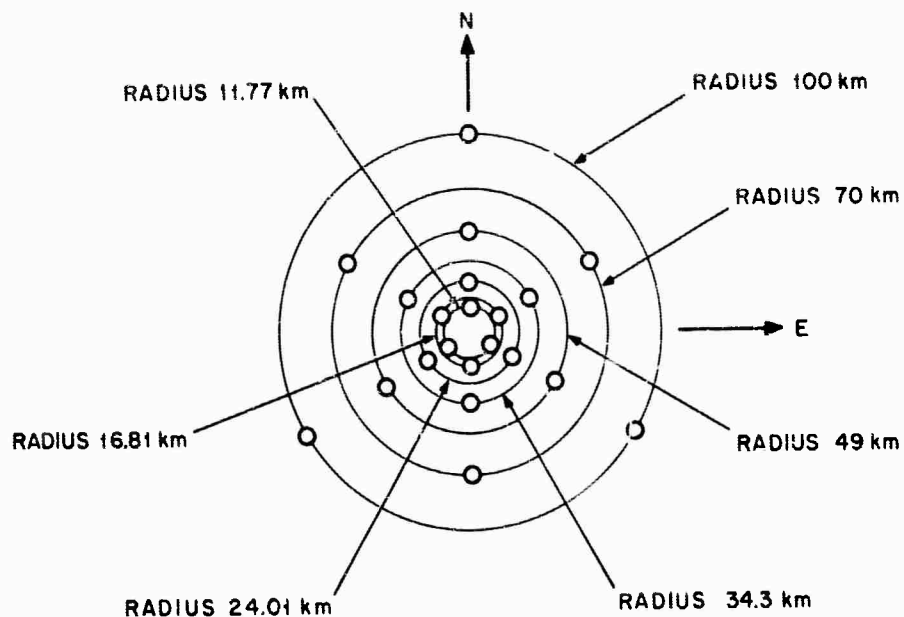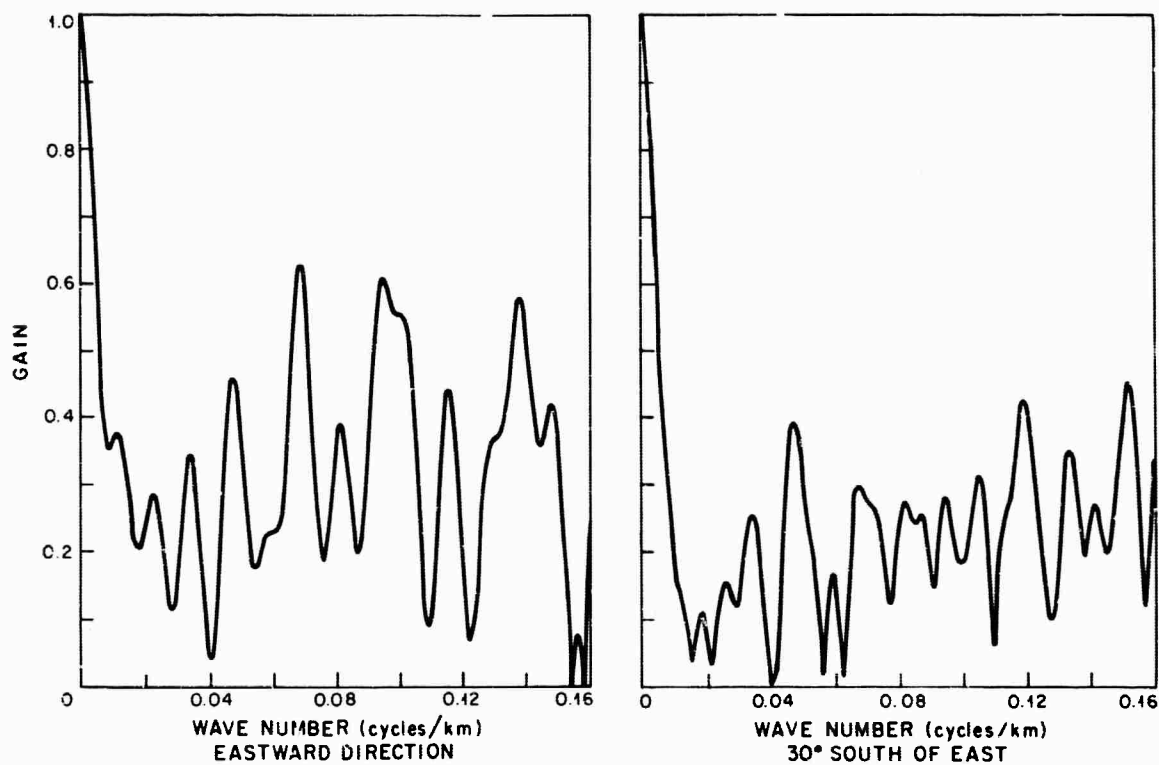
(B)   Pattern (cross sections).

Figure 14   Geometry and patterns for an array having four subarrays per
concentric ring.

(A)   Geometry having geometrical spacing of concentric rings.
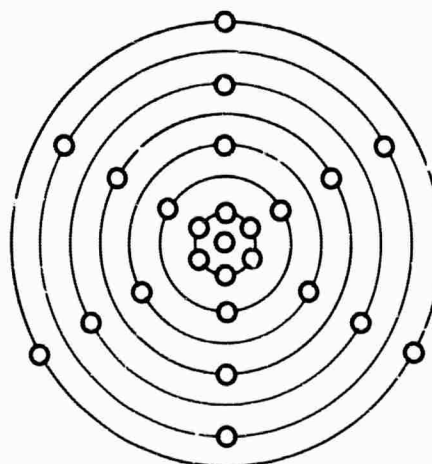
(B)   Geometry having arithmetical spacing of concentric rings.

Figure 15   Two array geometries which use 25 elements.

EAST k_x

NORTH k_y

.9 .7 .5 .3
.8 .6 .4

SCALE

0.01 cycles / km

3–64–4359
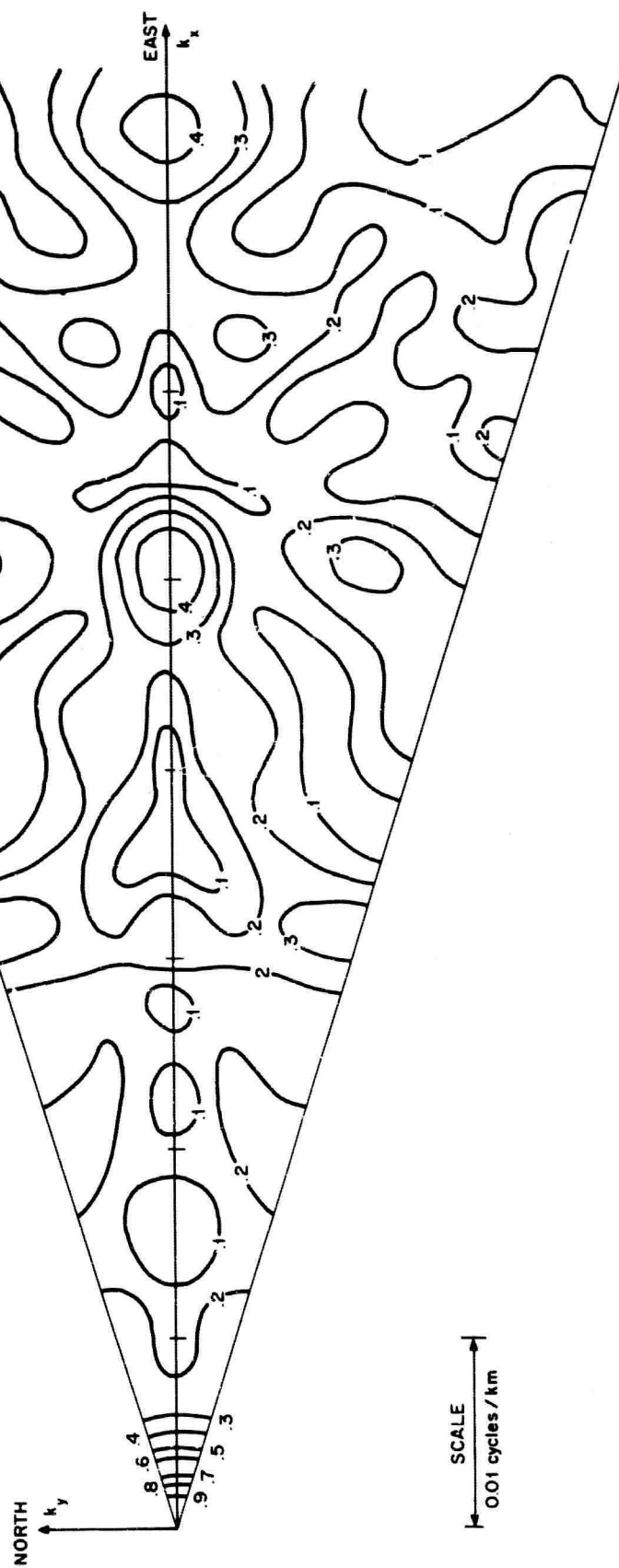
Figure 16   Pattern (contour map) for the array shown in Figure 15A.

73
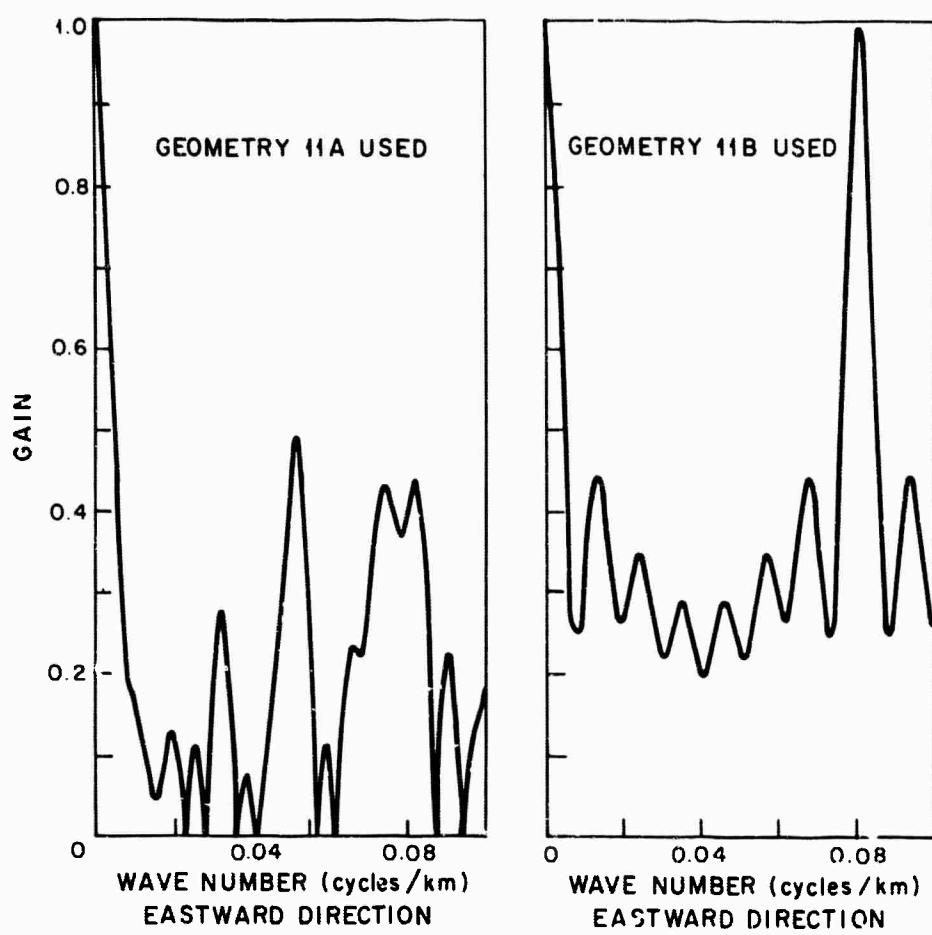
Figure 17   Patterns (cross sections) for both arrays shown in Figure 15.
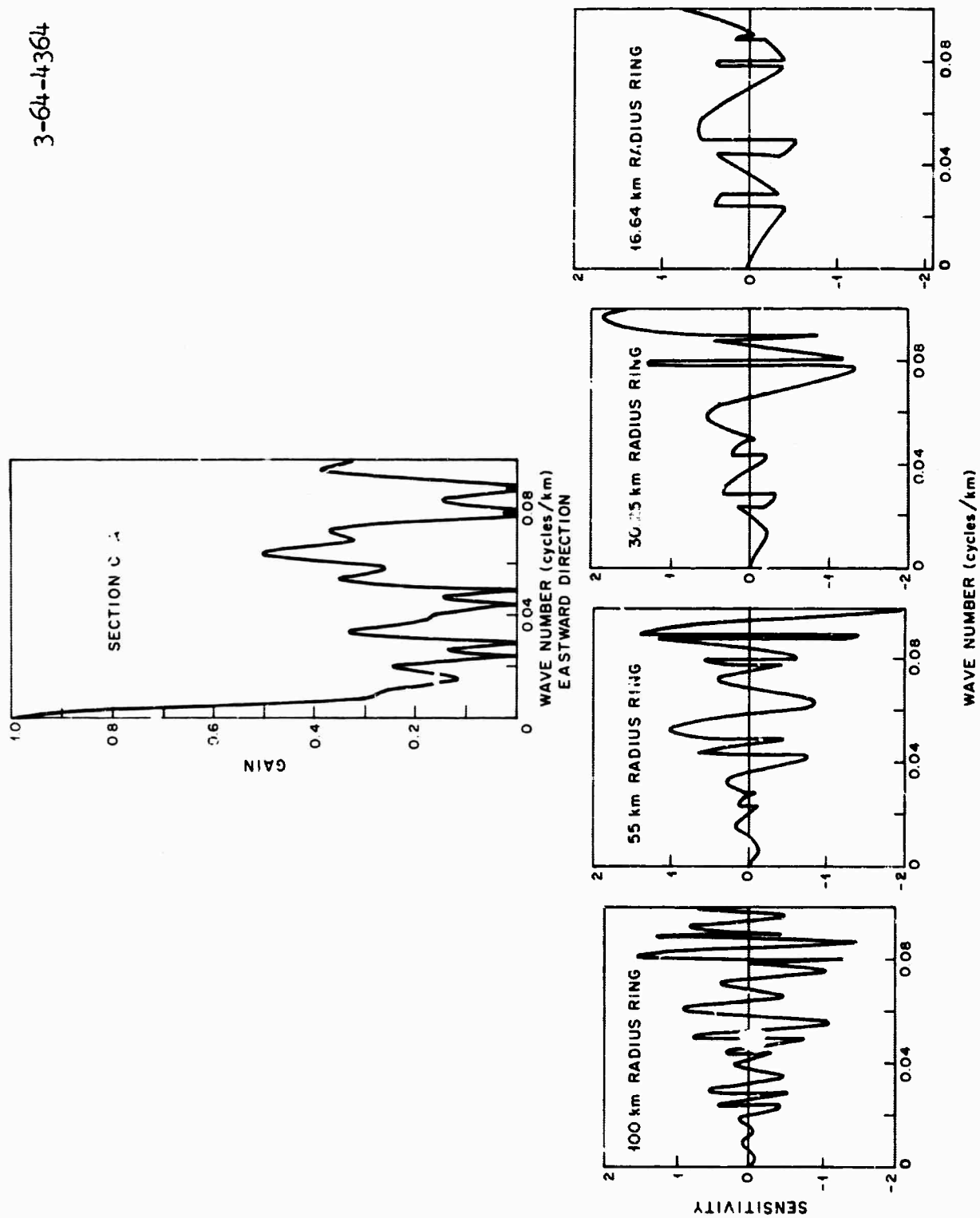
3-64-4361
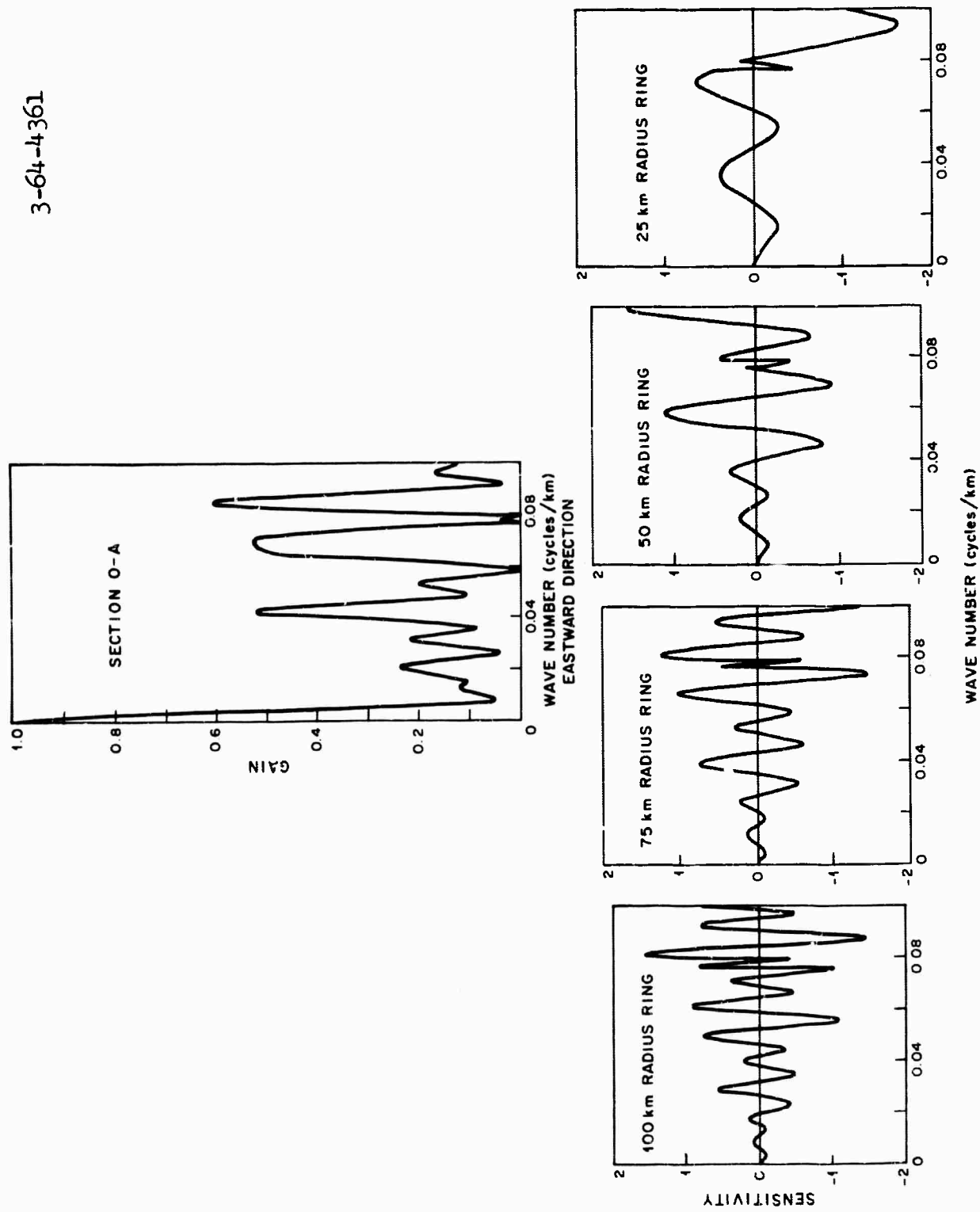
Figure 18   Pattern (cross section) and sensitivity functions for the array geometry shown in Figure 12A.

75
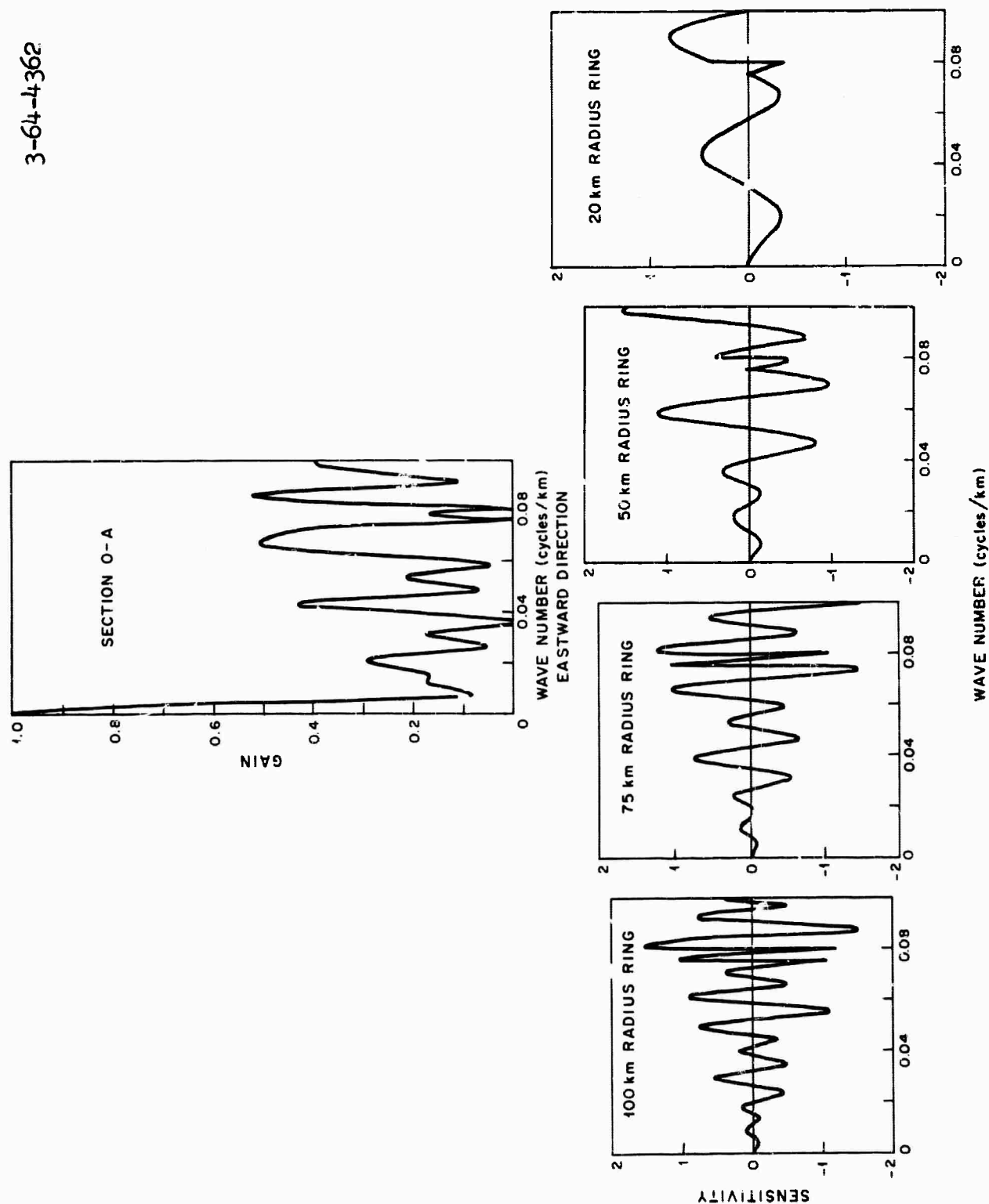
Figure 19 Pattern (cross section) and sensitivity functions for an array geometry which would be obtained from that of Figure 12A by reducing the 25 km ring to 20 km.
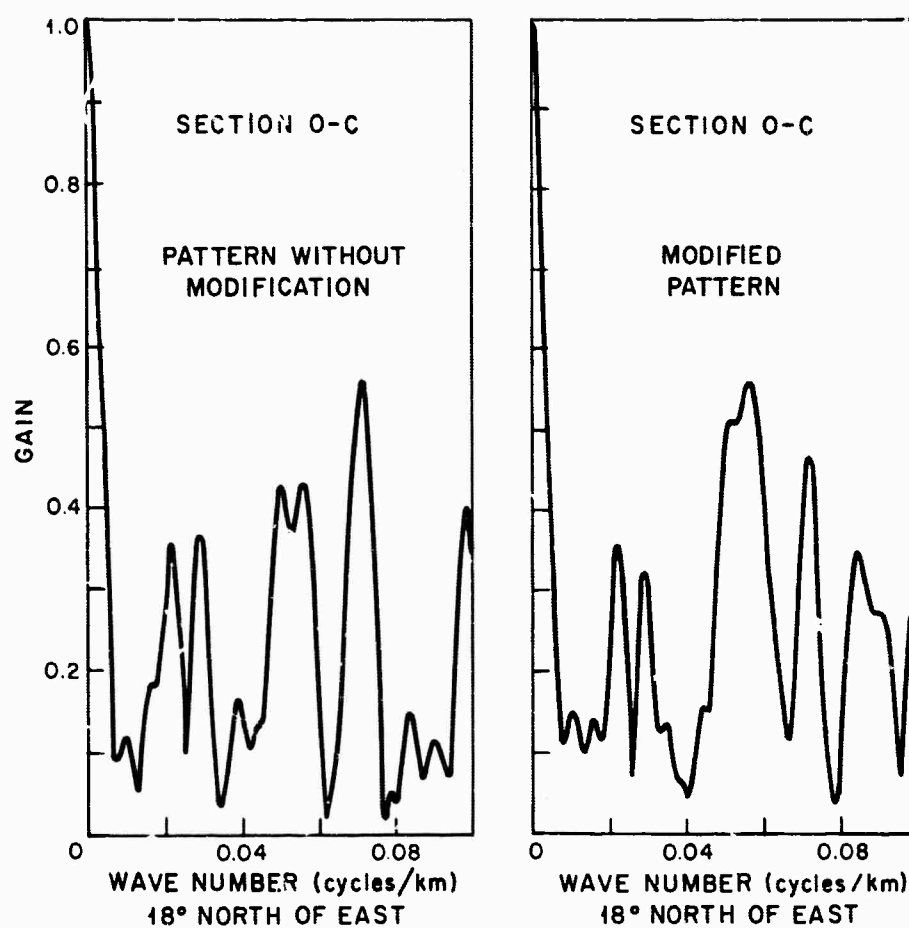
Figure 20   Patterns (cross sections in the direction 18° north of east in wave number space) showing some effects of reducing the 25 km ring of Figure 12A to 20 km.

77

Figure 21   Pattern (cross section) and sensitivity functions for the array geometry
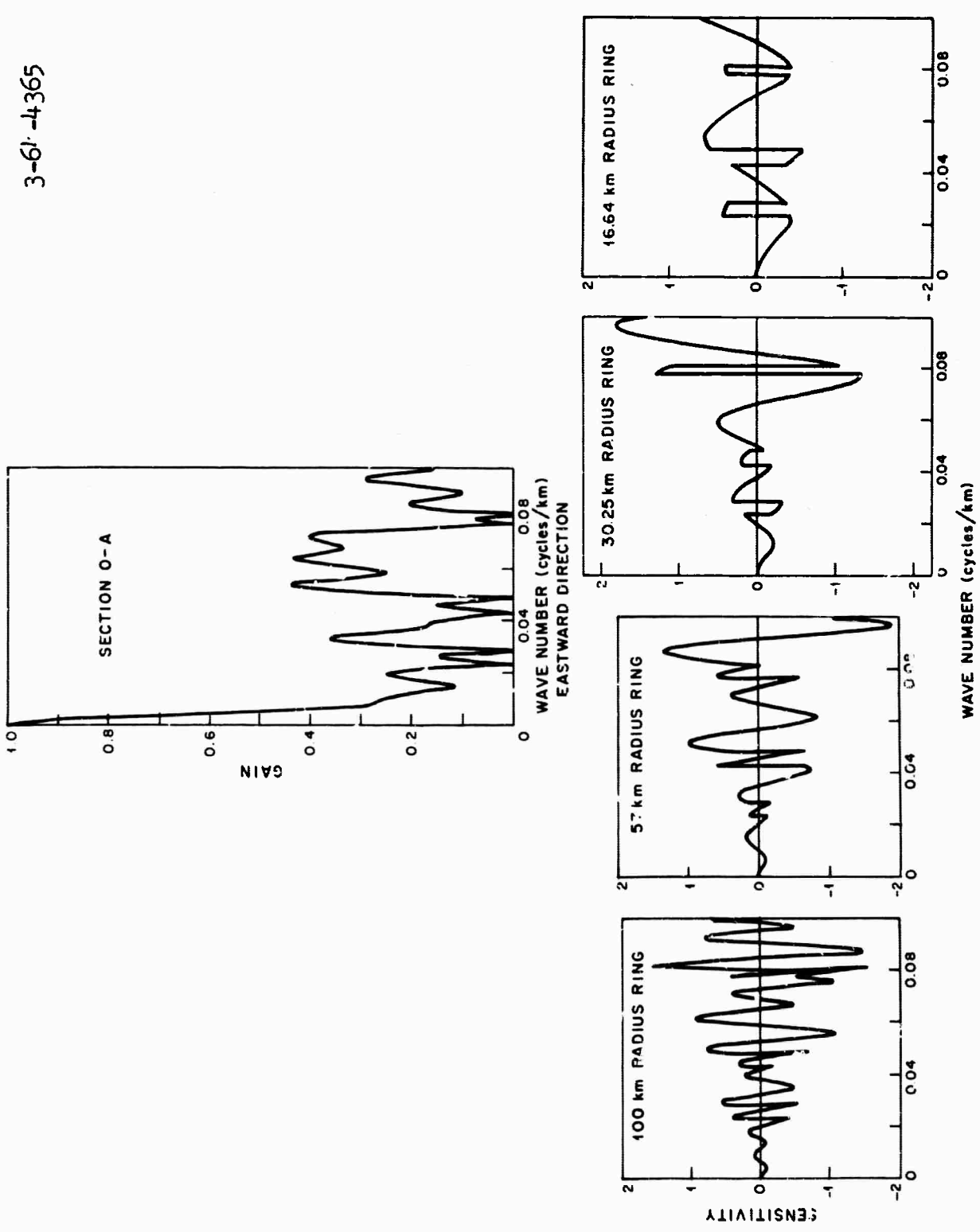shown in Figure 11A.

Figure 22   Pattern (cross section) and sensitivity functions for an array geometry which would be obtained from that of Figure 11A by increasing the 55 km ring to 57 km.
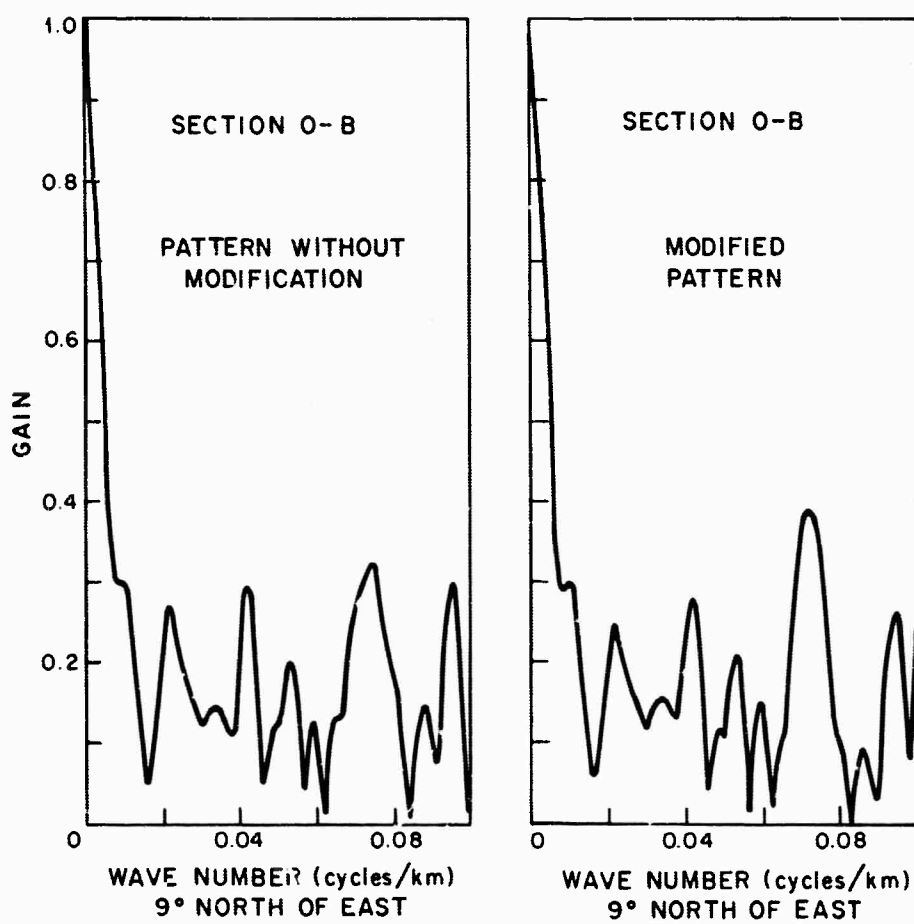
Figure 23   Pattern (cross sections in the direction 9° north of east in wave number space) showing some effects of increasing the 55 km ring of Figure 11A to 57 km.
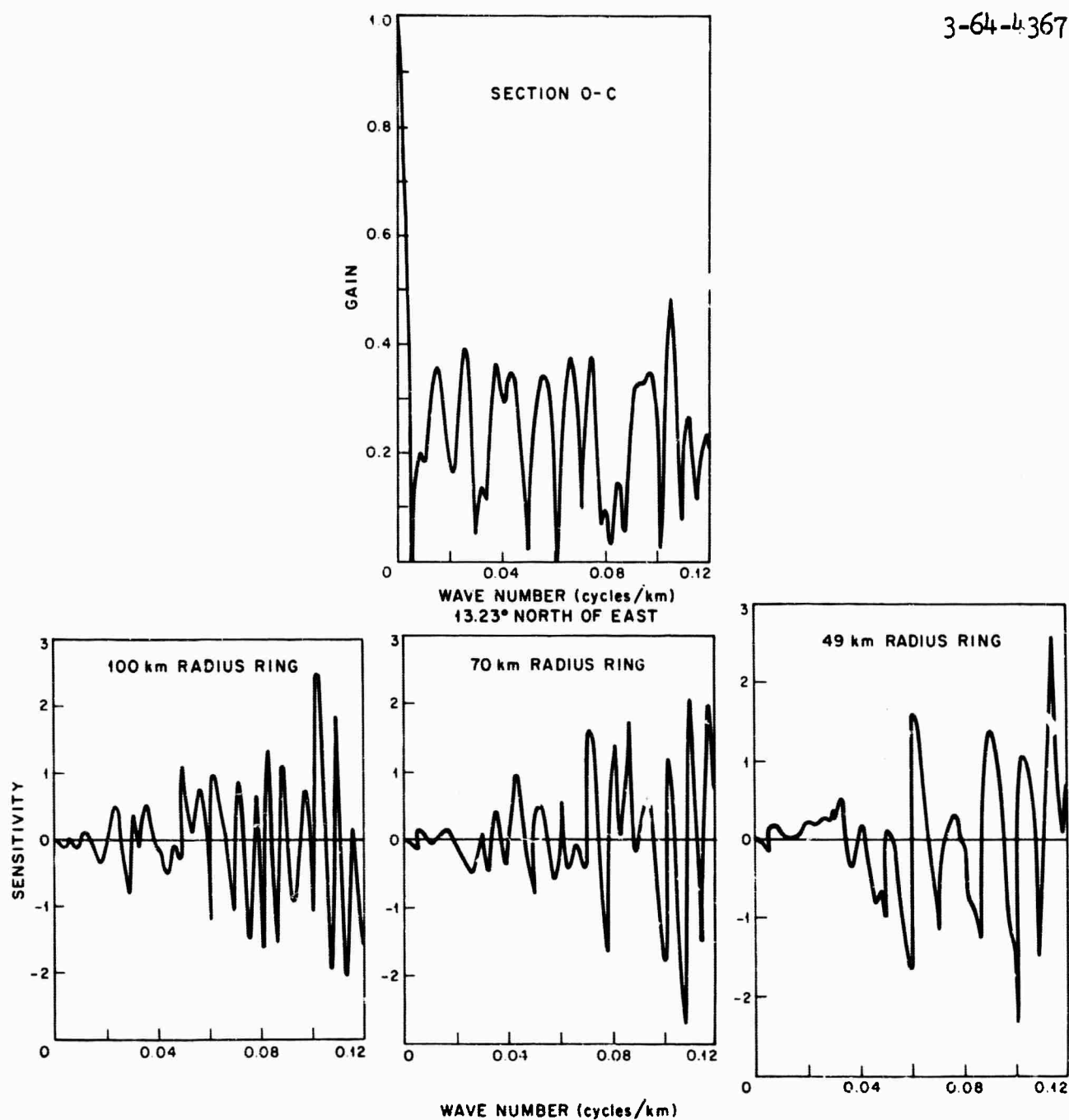
Figure 24   Pattern (cross section) and sensitivity functions for the array geometry shown in Figure 9A.
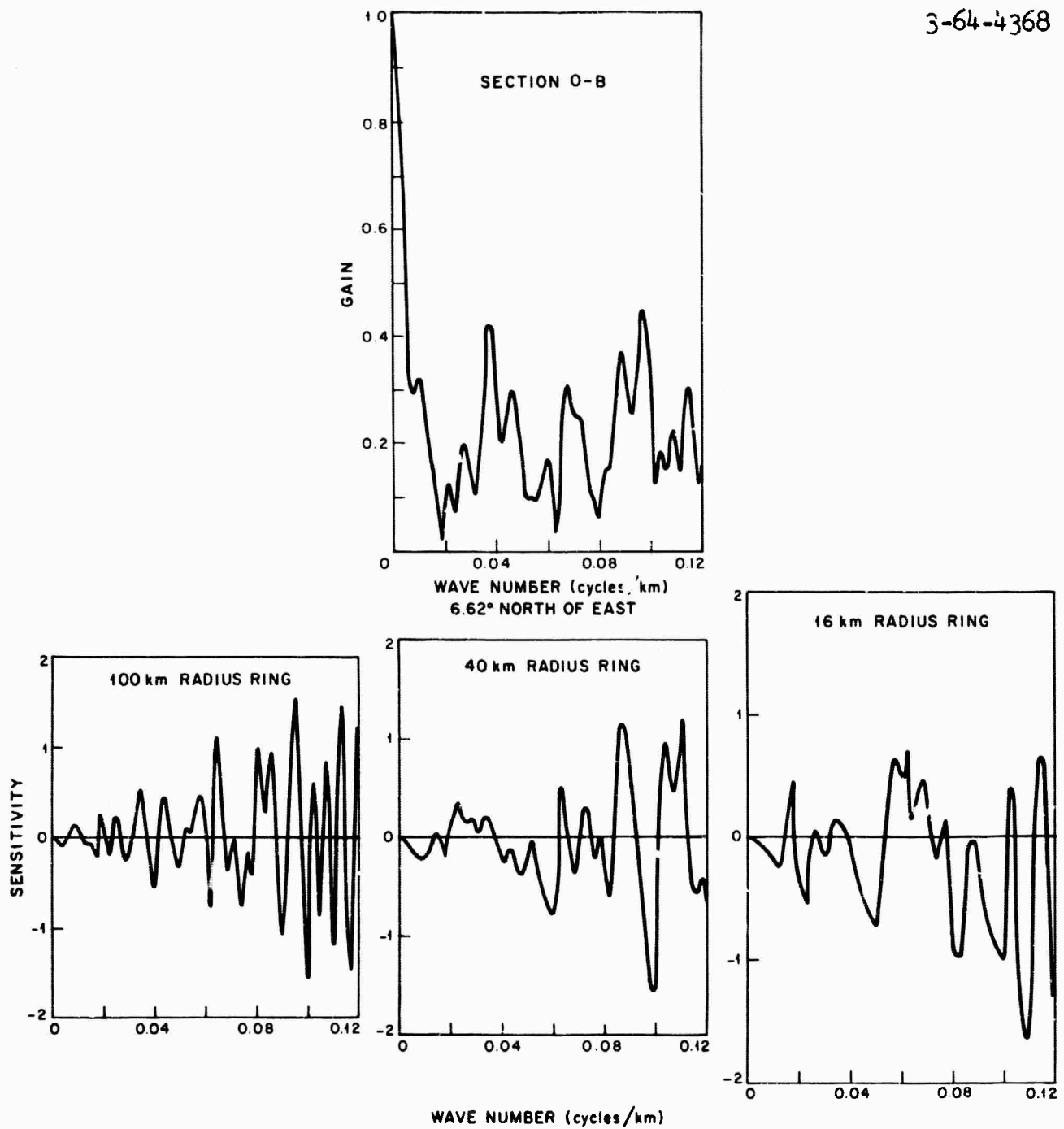
Figure 25   Pattern (cross section) and sensitivity functions for the array geometry shown in Figure 10A.

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13. ABSTRACT

A study of possible configurations for a Large Aperture Seismic Array has been completed. An array having 21 identical subarrays located on three concentric circles has been found to yield the most satisfactory pattern in wave number space of all the configurations tested. Patterns for some alternative placements of subarrays, including that of the experimental LASA in Montana, have been included in this report.

This study of patterns in wave number space has yielded the suggestion that a LASA having a diameter of 200 km should be composed of subarrays from 10 to 15 km in diameter. Such an increase of subarray size above the 7 km diameter subarrays of the experimental LASA would require the use of less regular subarray geometries than those which have been used in Montana.

A sensitivity function for patterns has been developed. This function can be used to predict the change in patterns which might result from changes in seismometer or subarray positions. Since the sensitivity function predicts possible changes in patterns, it can be used to set bounds upon changes in an array which can be made without severly changing the pattern. The tight bounds imposed by the sensitivity function can be relaxed if the pattern resulting from any anticipated change in position is actually computed. Since very little computer time is required to compute a pattern, this procedure is highly recommended.

14. KEY WORDS

seismic array	sensitivity function
array geometries	wave number space